HOLOGRAPHIC DARK ENERGY AND BRANS-DICKE THEORY

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Plan of the Talk

- Introduction
- Holographic Dark Energy Model
- Observational Constraints
- Present Acceleration
- Coincidence Problem
- Conclusion

Brans-Dicke Theory

For a FRW universe filled with perfect fluid, the gravitational field equations are

$$\frac{\dot{a}^2 + k}{a^2} + \frac{\dot{a}\dot{\phi}}{a\phi} - \frac{\omega\dot{\phi}^2}{6\phi^2} = \frac{8\pi\rho}{3\phi}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + 2\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi p}{\phi}$$

The wave equation for scalar field is

$$\frac{\ddot{\phi}}{8\pi} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{8\pi} = \frac{\rho - 3p}{2\omega + 3}$$

Our Assumption

- Flat FRW Universe
- Filled with dust and dark energy
- BD field varies with time as a power law of scale factor. i.e.

$$\Phi(t) \propto a(t)^n \qquad \textbf{(1)}$$

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The gravitational field equation becomes

$$3M_P^2\Phi\left[H^2+H\frac{\dot{\Phi}}{\Phi}-\frac{\omega}{6}\frac{\dot{\Phi}^2}{\Phi^2}\right]=\rho_x+\rho_m \qquad \textbf{(2)}$$

$$2\frac{\ddot{a}}{a}+H^2+\frac{\omega}{2}\frac{\dot{\Phi}^2}{\Phi^2}+2H\frac{\dot{\Phi}}{\Phi}+\frac{\ddot{\Phi}}{\Phi}=-\frac{p_x}{M_P^2\Phi} \qquad \textbf{(3)}$$

The wave equation for scalar field is

$$M_P^2\left(\ddot{\Phi}+3H\dot{\Phi}\right)=\frac{\rho_x+\rho_m-3p_x}{2\omega+3} \qquad \textbf{(4)}$$

The energy conservation equation becomes

$$\left(\dot{\rho}_x+\dot{\rho}_m\right)+3H\left(\rho_x+\rho_m+p_x\right)=0 \qquad \textbf{(5)}$$

Dark Energy Model

- Holographic Principle

$$\rho_x = 3c^2 M_P^2 L^{-2} \qquad \textbf{(6)}$$

- Hubble horizon as IR cutoff

$$\rho_x = 3c^2 M_P^2 H^2 \qquad \textbf{(7)}$$

- Interaction of matter and dark energy

Energy conservation equation becomes

$$\left.\begin{aligned} \dot{\rho}_m + 3H\rho_m &= Q \\ \dot{\rho}_x + 3H(1+\alpha)\rho_x &= -Q \end{aligned}\right\} \quad \textbf{(8)}$$

where $Q = \Gamma\rho_x$ with $\Gamma > 0$ is the interaction rate

and $\alpha = \dfrac{p_x}{\rho_x}$ is the equation of state parameter of the dark energy

Equations (1) and (2) gives

$$\rho_x + \rho_m = 3M_P^2 \Phi H^2 \left[(n+1) - \frac{n^2 \omega}{6} \right] \qquad \textbf{(9)}$$

Equations (7) and (9) leads to

$$c^2 r = \Phi \left[(n+1) - \frac{n^2 \omega}{6} \right] - c^2 \qquad \textbf{(10)}$$

This equation gives a generic value for n as $n \sim -0.016$ with both c, Φ of $O(1)$ and $|\omega| \cong 10^4$

Using equations (7), (10) and first equation of (8), we get

$$\dot{\rho}_x = \rho_x \left[\frac{\Gamma - 3Hr - nH(1+r)}{r} \right] \qquad \textbf{(11)}$$

Comparing equation (11) with second equation of (8), we find

$$\alpha = \frac{n}{3}\left(1 + \frac{1}{r}\right) - \frac{\Gamma}{3H}\left(1 + \frac{1}{r}\right) \qquad \textbf{(12)}$$

Different forms of α

1.
$$\alpha = \frac{n}{3}\left(1+\frac{1}{r}\right) - \frac{\Gamma}{3H}\left(1+\frac{1}{r}\right) \qquad \textbf{(13)}$$

2. For $a(t) \propto t^{\beta}$

$$\alpha(t) = \frac{n}{3}\left(1+\frac{1}{r}\right) - \frac{\Gamma}{3\beta}\left(1+\frac{1}{r}\right)t \qquad \textbf{(14)}$$

3. In terms of redshift $\left(H = -\frac{\dot{z}}{1+z}\right)$

$$\alpha(z) = \frac{n}{3}\left(1+\frac{1}{r}\right) + \left(\frac{1+z}{\dot{z}}\right)\frac{\Gamma}{3}\left(1+\frac{1}{r}\right) \qquad \textbf{(15)}$$

Dark energy density parameter

It is defined as $\Omega_x = \dfrac{\rho_x}{\rho_c}$ with $\rho_x = \rho_x^0 f(z)$

and

$$f(z) = \exp\left[3\int_0^z \frac{1+\alpha(z')}{1+z'}dz'\right] \qquad \textbf{(16)}$$

But $\dfrac{1}{r} \sim (1+z)^n$. So

$$\Omega_x = \left[1 + \frac{\Omega_m^0}{\Omega_x^0}(1+z)^{|n|\left(1+\frac{1}{r}\right)} \exp\left\{\Gamma\left(t_0 - t\right)\left(1+\frac{1}{r}\right)\right\}\right]^{-1} \qquad \textbf{(17)}$$

Estimation of Γ

For $t \approx 0$ equation (17) gives $\Omega_x \approx 0$

Present observation tells for

$$t = t_0 = 14 \times 10^9 \, yr, \;\; \Omega_x \approx 0.7$$

We estimate

$$\Gamma \approx 5 \times 10^{-11} (yr)^{-1} \approx 0.7 \times H_0 \qquad \textbf{(18)}$$

Ω_x for different era

- For radiation dominated era($a(t) \propto t^{1/2}$)

$$\Omega_x = \left[1 + \frac{\Omega_m^0}{\Omega_x^0} (1+z)^{|n|\left(1+\frac{1}{r}\right)} \exp\left\{ \Gamma t_0 \left(1 - \frac{1}{(1+z)^2} \right) \left(1 + \frac{1}{r} \right) \right\} \right]^{-1} \quad \textbf{(19)}$$

- For matter dominated era($a(t) \propto t^{(2\omega+2)/(3\omega+4)} \approx t^{2/3}$)

$$\Omega_x = \left[1 + \frac{\Omega_m^0}{\Omega_x^0} (1+z)^{|n|\left(1+\frac{1}{r}\right)} \exp\left\{ \Gamma t_0 \left(1 - \frac{1}{(1+z)^{3/2}} \right) \left(1 + \frac{1}{r} \right) \right\} \right]^{-1} \quad \textbf{(20)}$$

Observational Constraints

- LSS Constraint

  For $1 < z < 3, \ \Omega_x < 0.5$

  our calculation gives $\Omega_x < 0.34$

- BBN Constraint

  For $z = 10^{10}, \ \Omega_x < 0.21$

  our calculation gives $\Omega_x < 0.23$

Present value of α and Ω_x

For $\beta = 1.01$ we finds $\alpha_0 = -0.79$

experimental value is $\alpha_0 \leq -0.72$

For $z = 0$ equation (17) gives

$$\Omega_x^0 + \Omega_m^0 = 1$$

Deceleration parameter

Equations (1), (3) and (7) gives

$$q = \frac{1}{n+2}\left[\frac{3\alpha(z)c^2}{\Phi} + \frac{n^2\omega}{2} + \left(n^2 + n + 1\right)\right] \qquad \textbf{(21)}$$

for distance past ($z >> 1$), $q \to 1.17$

for present era($z = 0$), $q \approx -0.025$

Transition redshift is $z_{q=0} \sim 0.32$

experimental value is $z_{q=0} \sim 0.5$

Coincidence Problem

From equation (8), one can get

$$\frac{\dot{r}}{r} = 3H\left[\alpha + \left(1 + \frac{1}{r}\right)\frac{\Gamma}{3H}\right] \qquad \textbf{(22)}$$

Our calculation gives

$$\left|\frac{\dot{r}}{r}\right|_0 = 2.0 \times 10^{-2} \times 3H_0$$

ΛCDM model predicts $\left|\frac{\dot{r}}{r}\right|_0 = 3H_0$

Conclusions

- Addresses Present Acceleration
- Satisfies LSS and BBN constraints
- Agrees with experimental values of equation of state parameter of dark energy and Transition redshift
- Softens Coincidence Problem

THANK YOU