Constraints on Cosmological constant due to Scale invariance

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- *PLAN OF TALK :*

  1. *Introduction to Scale invariance*
  2. *Motivation of Scale invariant standard Model*
  3. *our work*
  4. *Conclusion*

Global scale invariance:

$$\phi(x) \to \phi'(x') = \frac{1}{\Lambda}\phi(x) \, ; \qquad x^\mu \to x'^\mu = \Lambda x^\mu$$

$$d^4x \to d'^4x = \Lambda^4 d^4x$$

$$A_\mu(x) \to A'_\mu(x') = \frac{1}{\Lambda} A_\mu(x) \, ,$$

$$L_0 = \frac{1}{2} g^{\mu\nu} \frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} \, ,$$

$$L_0 \to \frac{1}{\Lambda^4} L_0 \, ,$$

$$S = \frac{1}{2}\int L_0 \, |g|^{\frac{1}{2}} \, d^4x \, ,$$

→ *Invariant*

And The Local Scale transformation can be written as product of two transformation .

$$x^{\mu} \rightarrow x'^{\mu} ,$$

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = \left[\frac{\partial x^{\rho}}{\partial x'^{\mu}}\right]\left[\frac{\partial x^{\sigma}}{\partial x'^{\nu}}\right] g_{\rho\sigma} ,$$ *-1 [G.C.T .]*

$$\phi(x) \rightarrow \phi'(x') = \phi(x) ,$$

$$A_{\mu}(x) \rightarrow A'_{\mu}(x') = \frac{\partial x'^{\rho}}{\partial x^{\mu}} A_{\rho}(x),$$

$$\otimes$$

$$x'^{\mu} = x^{\mu}, \quad g'_{\mu\nu} = \Lambda^{2} g_{\mu\nu} ,$$

$$g'^{\mu\nu} = \frac{1}{\Lambda^{2}} g^{\mu\nu} , \quad A'_{\mu} = A_{\mu}$$ *-2 [P.S.T.]*

Changes in Derivative of fields In Local Scale Invariance

Now since,

$$\partial\left(\frac{1}{\Lambda}\phi\right) \neq \frac{1}{\Lambda}\partial\left(\phi\right)$$

So,

$$\partial_{\mu}\left(\phi\right) \rightarrow \left(\partial_{\mu} - f S_{\mu}\right)\left(\phi\right) \ ,$$

$$S_{\mu} \rightarrow S_{\mu} - \frac{1}{f}\partial_{\mu}\left(\ln\Lambda\right) ,$$

Similarly the derivatives of other fields will also be changed.

So ,we have :

$$
\begin{aligned}
\partial_\mu(\phi) &\to \left(\partial_\mu - f S_\mu\right)(\phi)\ , \\
\partial_\mu(g_{\nu\rho}) &\to \left(\partial_\mu + 2 f S_\mu\right)\left(g_{\nu\rho}\right), \\
\partial_\mu(g^{\nu\rho}) &\to \left(\partial_\mu - 2 f S_\mu\right)\left(g^{\nu\rho}\right), \\
\partial_\mu(A_\nu) &\to \partial_\mu(A_\nu)\ .
\end{aligned}
$$

Motivation of scale invariant standard model:

1. *Scale invariance is one kind of symmetry.*
2. *A simple local scale invariant Phi-4 theory give the dynamical Explanation of dark matter.*
3. *It was shown by authors (a,b)that Scale symmetry is not anomalous.*

Scale Invariance removes the constants in usual standard model.

a: E. Englert ,C. Turffin and R. Gastmans (1976)
b: R. Kallosh (1975)

Our work:

The action for scale invariant extension of standard model in d dim:

$$\mathcal{S} = \int d^d x \sqrt{-\bar{g}} \Big[-\frac{\beta}{4} H^\dagger H \bar{R}' + \bar{g}^{\mu\nu} (D_\mu H)^\dagger (D_\nu H) - \frac{1}{4} \bar{g}^{\mu\nu} \bar{g}^{\alpha\beta} (\mathcal{A}^i_{\mu\alpha} \mathcal{A}^i_{\nu\beta}$$
$$+ \mathcal{B}_{\mu\alpha} \mathcal{B}_{\nu\beta} + \mathcal{G}^j_{\mu\alpha} \mathcal{G}^j_{\nu\beta}) (\bar{R}'^{\,2})^{-\epsilon/4} - \frac{1}{4} \bar{g}^{\mu\rho} \bar{g}^{\nu\sigma} \mathcal{E}_{\mu\nu} \mathcal{E}_{\rho\sigma} (\bar{R}'^{\,2})^{-\epsilon/4}$$
$$- \lambda (H^\dagger H)^2 (\bar{R}'^{\,2})^{\epsilon/4} \Big] + \mathcal{S}_{\mathrm{fermions}}$$

Where H is Higgs doublet ,

$\mathrm{G}^j_{\mu\nu}, \mathrm{A}^i_{\mu\nu}, \mathrm{B}_{\mu\nu} and \mathrm{E}_{\mu\nu}$ *are SU(3),SU(2),U(1) and Weyl vector fields res.*

$$\mathcal{S}_{\rm fermions} = \int d^dx\, e\, \left(\overline{\Psi}_L i\gamma^\mu \mathcal{D}_\mu \Psi_L + \overline{\Psi}_R i\gamma^\mu \mathcal{D}_\mu \Psi_R\right) - \int d^dx\, e\, (g_Y \overline{\Psi}_L \mathcal{H} \Psi_R (\bar{R}'^2)^{\epsilon/8} + h.c.), \quad (3)$$

where $e = \det(e_\mu{}^a)$, $e_\mu{}^a e_\nu{}^b \eta_{ab} = g_{\mu\nu}$, $\gamma^\mu = e^\mu{}_a \gamma^a$ and a, b are Lorentz indices. Here Ψ_L is an $SU(2)$ doublet, Ψ_R a singlet and g_Y represents a Yukawa coupling. For simplicity we have displayed only one Yukawa coupling term. Furthermore we have displayed the action only for a single family.

The covariant derivative acting on the fermion field is defined by

$$\mathcal{D}_\mu \Psi_{L,R} = \left(\tilde{D}_\mu + \frac{1}{2}\omega_\mu^{ab}\sigma_{ab}\right)\Psi_{L,R}\ , \quad (4)$$

where $\tilde{D}_\mu \Psi_L = \partial_\mu - ig\mathbf{T}\cdot\mathbf{A}_\mu - ig'\frac{Y_f^L}{2}B_\mu$, $\tilde{D}_\mu \Psi_R = \partial_\mu - ig'\frac{Y_f^R}{2}B_\mu$ and $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$. In Eq. 4, A_μ is the $SU(2)$ field, B_μ the $U(1)$ field, $\mathbf{T}$ represents the SU(2) generators and Y_f the U(1) hypercharges. Here we have not explicitly displayed the color interactions for quarks, which can be easily added. The spin connection ω_μ^{ab} can be solved in terms of the vierbein,

$$\omega_{\mu ab} = \frac{1}{2}(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu})e_a{}^\nu - \frac{1}{2}(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu})e_b{}^\nu - \frac{1}{2}e_a{}^\rho e_b{}^\sigma(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho})e^c{}_\mu\ . \quad (5)$$

The spin connection term in covariant derivative makes the fermionic action locally scale invariant. The Weyl vector boson does not couple directly to fermions. However it couples indirectly since the factor $\bar{R}'$ contains contribution from the Weyl meson.

Constraints on the Cosmological Constant

1. Pure Gauge Fields:

$$\mathcal{S}_{\rm A} = \int d^d x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\nu} g^{\alpha\beta} (\mathcal{A}^i_{\mu\alpha} \mathcal{A}^i_{\nu\beta}) (R^2)^{-\epsilon/4} \right],$$

$$T^{\rm A}_{\sigma\rho} = \frac{1}{4} g_{\sigma\rho} [g^{\mu\nu} g^{\alpha\beta} A^i_{\mu\alpha} A^i_{\nu\beta} (R^2)^{-\epsilon/4}] - [g^{\alpha\beta} A^i_{\sigma\alpha} A^i_{\rho\beta} (R^2)^{-\epsilon/4}]$$
$$+ \frac{\epsilon}{4} [g^{\mu\nu} g^{\alpha\beta} A^i_{\mu\alpha} A^i_{\nu\beta} (R^2)^{-(\epsilon+2)/4}] R_{\sigma\rho}$$
$$+ \frac{\epsilon}{4} [g^{\mu\nu} g^{\alpha\beta} A^i_{\mu\alpha} A^i_{\nu\beta} (R^2)^{-(\epsilon+2)/4}]_{;\gamma;\delta} \left[-\frac{1}{2} (g^\gamma_\sigma g^\delta_\rho + g^\delta_\sigma g^\gamma_\rho) + g_{\sigma\rho} g^{\gamma\delta} \right].$$

$$\frac{\epsilon}{4}[g^{\mu\nu} g^{\alpha\beta} A^i_{\mu\alpha} A^i_{\nu\beta} (R^2)^{-(\epsilon+2)/4}]^{;\gamma}_{;\gamma} (d-1)$$

(a) The kinetic energy terms for the vector fields do not contribute to Cosmological constant.

(b) The results applies directly to QED and QCD gauge fields and in exact at all orders in the gauge coupling. Here higher order in gravitational coupling and Weyl meson coupling f has not been considered.

2. Mass less Dirac fermions:

$$\mathcal{S}_\Psi = \int d^dxe\left(\overline{\Psi}i\gamma^\mu\mathcal{D}_\mu\Psi\right).$$

$$T^\Psi_{\alpha\beta} = -g_{\alpha\beta}(\overline{\Psi}i\gamma^\rho\mathcal{D}_\rho\Psi) + \frac{1}{2}\overline{\Psi}i\gamma_\alpha\mathcal{D}_\beta\Psi + \frac{1}{2}\overline{\Psi}i\gamma_\beta\mathcal{D}_\alpha\Psi$$
$$+ \frac{g_{\alpha\beta}}{2}(\overline{\Psi}i\gamma_\rho\Psi)^{;\rho} - \frac{1}{4}(\overline{\psi}i\gamma_\beta\psi)_{;\alpha} - \frac{1}{4}(\overline{\psi}i\gamma_\alpha\psi)_{;\beta}.$$

$$i\gamma^\mu\mathcal{D}_\mu\Psi = 0$$

*The above result also applies directly if we include gauge interactions of fermions.
We find that we get zero contribution to the cosmological constant from both
The strong field and electromagnetic interactions as long as we do not include
The mass term for fermions or equivalently Yukawa interactions terms.*

3. Pure Higgs fields:

$$T_{\alpha\beta} = (D_\alpha H)^\dagger D_\beta H + (D_H)^\dagger D_H - g_{\alpha\beta}\left[g^{\mu\nu}(D_\mu H)^\dagger D_\nu H - \lambda(H^\dagger H)^2(R^2)^{\epsilon/4}\right]$$
$$- (H^\dagger H)^2(R^2)^{(-2)/4}R + \ldots .$$

$$T^\alpha_\alpha = \left(1 - \frac{\epsilon}{2}\right)\left[-2D_\mu(g^{\mu\nu}H^\dagger D_\nu H) + 2g^{\mu\nu}H^\dagger D_\mu D_\nu H + 4\lambda(H^\dagger H)^2(R^2)^{\epsilon/4}\right] +$$
$$\ldots .$$

With help of following Einstein Equation

$$H^\dagger H\left[-\frac{1}{2}g_{\alpha\beta}R + R_{\alpha\beta}\right] + (H^\dagger H)_{;\lambda;\kappa}\left[-\frac{1}{2}(g^\lambda_\alpha g^\kappa_\beta + g^\kappa_\alpha g^\lambda_\beta) + g_{\alpha\beta}g^{\lambda\kappa}\right] = \frac{2}{\beta}T_{\alpha\beta}$$

We get:

$$H^\dagger D^\mu D_\mu H + \frac{\beta}{4} R H^\dagger H + 2\lambda (H^\dagger H)^2 (R^2)^{\epsilon/4} + \ldots = 0\,,$$

$$D^\mu D_\mu H + \frac{\beta}{4} R H + 2\lambda H (H^\dagger H)(R^2)^{\epsilon/4} = 0\,,$$

Here we find two above equation are consistent with one another.

The Scale Invariant Standard Model

considering the full action the equation of motion of scalar field:

$$D_\mu D^\mu H + 2\lambda(H^\dagger H)H(R^2)^{\epsilon/4} + g_Y\bar{\Psi}_{\rm R}\Psi_{\rm L}(R^2)^{\epsilon/8} + \frac{\beta}{4}HR = 0$$

……………………………..(X)

The equation of motion of SU(2) doublet and singlet fermions:

$$i\gamma^\mu D_\mu\Psi_{\rm R} - g_Y H^\dagger\Psi_{\rm L}(R^2)^{\epsilon/8} = 0,$$
$$i\gamma^\mu D_\mu\Psi_{\rm L} - g_Y H\Psi_{\rm R}(R^2)^{\epsilon/8} = 0.$$

The energy mom. :

$$
\begin{aligned}
T_{\alpha\beta} = -g_{\alpha\beta}\Bigg[&(D_\mu H)^\dagger D^\mu H + \bar{\Psi}_L i\gamma^\rho D_\rho \Psi_L + \bar{\Psi}_R i\gamma^\rho D_\rho \Psi_R - (H^\dagger H)^2 (R^2)^{\epsilon/4} \\
&- g_Y\big(\bar{\Psi}_{\mathrm{L}} H \Psi_{\mathrm{R}} + h.c.\big)(R^2)^{\epsilon/8}\Bigg] + (D_\alpha H)^\dagger D_\beta H + (D_H)^\dagger D_H \\
&+ \frac{1}{2}\bar{\Psi}_L i\gamma_\alpha D_\beta \Psi_L + \frac{1}{2}\bar{\Psi}_R i\gamma_\alpha D_\beta \Psi_R + \frac{1}{2}\bar{\Psi}_L i\gamma_{D\Psi L} + \frac{1}{2}\bar{\Psi}_R i\gamma_{D\Psi R} \\
&- \epsilon\big(H^\dagger H\big)^2 (R^2)^{(\epsilon-2)/4} R_{\alpha\beta} - \frac{\epsilon g_Y}{2}\big(\bar{\Psi}_{\mathrm{L}} H \Psi_{\mathrm{R}} + h.c.\big)(R^2)^{(\epsilon-4)/8} R_{\alpha\beta} + \ldots,
\end{aligned}
$$

Simplified energy momentum tensor (using fermion's equations of motion To eliminate the kinetic energy term)

$$T^{\alpha}_{\alpha} = -(1-\epsilon/2)\left[2(D_{\mu}H)^{\dagger}D^{\mu}\Phi - g_Y(\bar{\Psi}_{\rm L}H\Psi_{\rm R} + h.c.)(R^2)^{\epsilon/8} - 4\lambda(H^{\dagger}H)^2(R^2)^{\epsilon/4}\right] + \dots,$$

Using following Einstein equation and above equation we find equation – (X.) So the equations are consistent to each other.

$$H^{\dagger}H\left[-\frac{1}{2}g_{\alpha\beta}R + R_{\alpha\beta}\right] + (H^{\dagger}H)_{;\lambda;\kappa}\left[-\frac{1}{2}(g^{\lambda}_{\alpha}g^{\kappa}_{\beta} + g^{\kappa}_{\alpha}g^{\lambda}_{\beta}) + g_{\alpha\beta}g^{\lambda\kappa}\right] = \frac{2}{\beta}T_{\alpha\beta}$$

In this model the cosmological constant is generated by the phenomenon of cosmological symmetry breaking. we assume a FRW metric with curvature parameter $k = 0$ and we expand the scalar field around its classical solution, $\mathcal{H} = \mathcal{H}_0 + \eta$, with,

$$\mathcal{H}_0^\dagger \mathcal{H}_0 = -\frac{\beta R}{4\lambda}. \tag{38}$$

Here we assume that $< S_\mu >= 0$, i.e. the expectation value of the Weyl meson field is identically equal to zero. Taking one point function, $< \eta >$, must vanish in this theory. This implies that

$$< \mathcal{H} >= \mathcal{H}_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{39}$$

exactly to all order in the perturbation theory. Here

$$v = \sqrt{-\frac{\beta R}{4\lambda}}\,. \tag{40}$$

The parameter $\mathcal{H}_0$ in Eq. 39 is fixed by its relationship to the W boson mass,

$$M_{\rm W}^2 = g^2(\mathcal{H}_0^\dagger \mathcal{H}_0), \tag{41}$$

where g is the gauge coupling. In Eq. 39 we have used the electroweak symmetry to set the first entry of the classical scalar doublet to be zero. This is analogous to the choice normally made in analysing the electroweak symmetry breaking in the standard Weinberg-Salam model. We, therefore, find that the curvature scalar and hence the effective cosmological constant is determined by Eqs. 39 and 40 in the standard model with local scale invariance. In Eq. 40, β and λ are the renormalized parameters. The parameter $1/\beta$ is effectively the gravitational coupling. Classically it is related to the Planck mass by the formula,

$$\beta \mathcal{H}_0^\dagger \mathcal{H}_0 = \frac{M_{\rm PL}^2}{4\pi}\,. \tag{42}$$

Hence this parameter is fixed by the value of the known gravitatonal coupling.

Conclusion:

1. *There are no contributions of pure gauge fields and mass less fermions in Cosmological constant.*

2. *Cosmological symmetry breaking generate the cosmological constant .*
3. *This Symmetry Breaking also generate the masses of massive fields , besides the Planck mass.*

Thank you………………..

$$\phi = \exp(i\theta)|\phi|;$$

$$\theta = \theta + \chi$$

$$\phi = \exp\left(i(\theta - i\ln|\phi|)\right);$$

Back