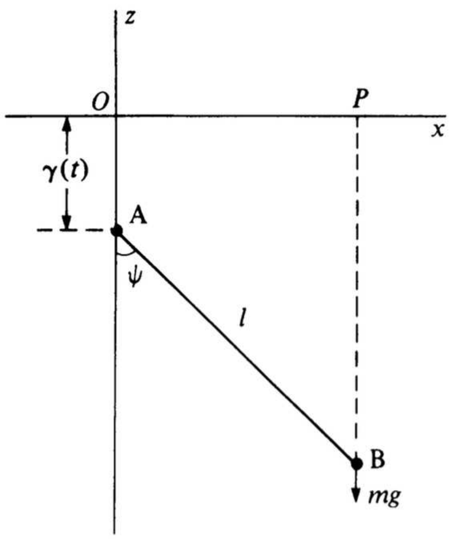
Ph. D Written Examination, 8/12/15. Time 2 hrs, Marks 100.

Please Answer ALL Questions SERIALLY.

1) Consider that in a three dimensional Euclidean space, a pendulum comprising



a mass m attached to a light, stiff rod AB of length l is free to move in a vertical plane. The end A of the rod is forced to move vertically, its distance from a fixed point O being a given function $\gamma(t)$ of time. The mass is at point B. Refer to the figure for a schematic diagram of the system.

(a) What is the number of degrees of freedom? [2]

(b) Write the equations of constraints explicitly. [4]

(c) Find the Lagrangian and the Hamiltonian of the system. [8+6]

2) A particle in a potential well $U(x)$ is initially in a state whose wavefunction $\Psi(x,0)$ is an equal-weight superposition of the ground state (with wavefunction ψ_0, and energy E_0) and first excited state (with wavefunction ψ_1, and energy E_1):

$$\Psi(x,0) = C(\psi_0(x) + \psi_1(x)).$$

(a) Obtain the normalization C for Ψ, assuming ψ_0 and ψ_1 to be normalized. [2]

(b) Determine $\Psi(x,t)$ at any later time t. [2]

(c) Calculate the average energy $< E >$ for $\Psi(x,t)$. [3]

(d) Determine the uncertainty ΔE of energy for $\Psi(x,t)$. [6]

(e) Determine the average position $< x(t) >$ of a particle with non-stationary state wave function $\Psi(x,t)$. [4]

(f) Plot the time-dependence obtained above, clearly identifying all the parameters involved. [3]

3) Consider a system of isolated N non-interacting particles. Each particle can have only of the two energy levels $-\epsilon_0$ and ϵ_0.
(a) Given that the total energy of the system is $M\epsilon_0$ (where M is an integer $M = -N, \cdots + N$), find out the number of accessible of micro states and hence the entropy. Using Stirling approximation ($\log n! = n \log n - n$), find an expression of temperature (T) as a function of M and N. [2+3]
(b) What happens to temperature when $M > 0$? [2]
(c) Considering the situation $M < 0$, find out the number of particles in the level ϵ_0 in terms of N, ϵ_0 and T and hence derive an expression for energy (E). Plot E as a function of T clearly showing the limits $T \to 0$ and $T \to \infty$. [2+3+3]
(d) Derive an expression for the specific heat per particle and plot in variations as a function of temperature specially clearly showing the limits $T \to 0$ and $T \to \infty$. Do you expect a maximum at some temperature? [5]

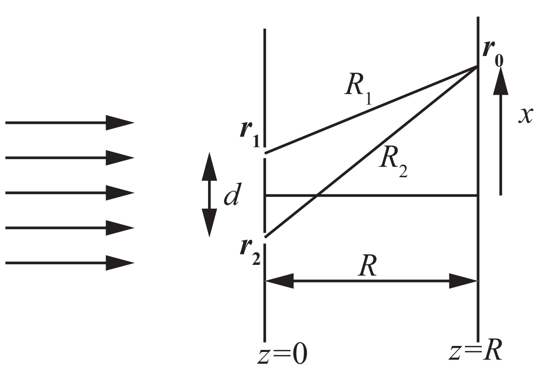
4a)A model for the electrostatic potential of an atom, due to the nucleus (charge $+Ze$) and electrons, is the so-called "screened Coulomb potential" is given by

$$V(r) = A\frac{e^{-\lambda r}}{r},$$

where $A = Ze/(4\pi\epsilon_0)$, $1/\lambda$ is an effective atomic radius and $r = |\mathbf{r}|$. Find the electric field $\mathbf{E}(r)$, the charge density $\rho(r)$ and the total charge Q. Sketch $\rho(r)$ as a function of r. [3+3+4+2]

4b) Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω and phase angle zero that is (i) travelling in the negative x direction and polarized in the z direction and (ii) travelling in the direction from the origin to the point (1,1,1) with the polarization parallel to the xz plane. [4+4]

5) (a) Let us consider the Young's double-slit experiment. Assume that the field



amplitude of the incoming field at $z = 0$ is given by $E(\vec{\mathbf{r}}, t) = Ae^{i(kz-\omega t)}$, where

$k = \frac{2\pi}{\lambda}$ is the wave-vector and $\omega = 2\pi\nu$ is the angular frequency of the incoming wave. Assuming $x \gg R$ and $d \gg R$, derive an expression for the intensity at $z = R$ as a function x, d, k and R. [6]

(b) Assume $R = 1$ m, $d = 1$ mm. Find the fringe period when the incident field on the double-slit has a wavelength $\lambda = 5000$ Å. [2]

(c) Given a slit-separation d, state what is the most essential characteristic that the incident field must have in order to produce high-visibility interference pattern at $z = R$. [2]

(d) Using complex analysis, evaluate $\int_0^\infty \frac{dx}{x^4+1}$ [4]

(e) Assuming an ideal opamp, find the expression for output voltage (V_o) in the following circuit. [6]