Ph.D Selection Test

Department of Physics

Indian Institute of technology Kanpur

May 14, 2019 Time: 9:30AM − 11:30AM Full Marks: 70

Answer all parts of a question in one place.

Question 1

Consider a free particle wave function in one dimension is given by

$$\psi(x) = A\ exp\{-\frac{1}{2}\frac{(x-a)^2}{x_0^2}\},$$

where a and x_0 are constants. Find $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, $\langle p^2\rangle$ and the uncertainty product. [5]

Question 2

(a) Find the eigenvalues of the operator $\hat{A} = 2\hat{J}_x + 2\hat{J}_y + \hat{J}_z$, where the operators denote the components of the angular momentum operator with a magnitude $J = 2$. [2]

(b) Consider a state $\mid \psi\rangle = a \mid 2,2\rangle + b \mid 2,0\rangle + c \mid 2,-2\rangle$ wiritten in $\mid j,m\rangle$ basis. Find the expectation value $\langle\psi \mid \hat{A} \mid \psi\rangle$. [3]

Question 3

Find the singularities for the following complex functions. In case there is a branch cut, show it by a suitable sketch. [2+2]

(a) $\sec(z)$

(b) $\log(z^2 - 3z + 2)$

Question 4

Find the Greens function for an oscillator obeying:

$$\ddot{Q}(t) + \eta\dot{Q}(t) + \Omega^2 Q(t) = 0.$$

In the above, $\dot{Q}(t) = \frac{dQ(t)}{dt}$. [6]

Question 5

(a) Consider two energy levels with energies ϵ and $-\epsilon$. Total N classical distinguishable particles can occupy these levels, where the positions of these particles are fixed. If the total energy is E, write an expression for the number of configuration possible in terms of N, E and ϵ. [3]

(b) Considering N, N_+ , $N_- \gg 1$ (where $N_\pm$ are number of particles in $\pm\epsilon$ state), write an expression of the entropy in terms of N, E and ϵ. (Hint: for a large number p, $ln(p!) \approx ln(p) - p$.) [4]

(c) Show that, the entropy from (b) is extensive (E is also extensive). [3]

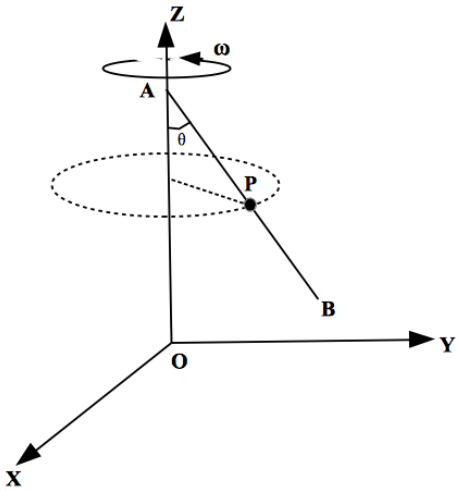
Question 6

A bead of mass m is constrained to move along a massless and frictionless rod (AB) fixed at a constant angle θ with the vertical axis as shown in the figure. The rod rotates with a uniform angular speed ω. At a time t, the distance of the instantaneous position P of the bead from A is denoted by r. The distance OA is h.

(i) Set up the Lagrangian for the bead in terms of given variables assuming at $t = 0$, the rod is in XZ plane. (Direct writing of the expression will be awarded zero credit). [4]

(ii) Using Euler-Lagrange equation, find the equation of motion of the bead. [2]

(iii) Solve for r at any arbitrary time t assuming that the bead was initially at rest and was situated at A. [4]



Question 7

(a) Write down the Maxwell equations for electrodynamics (in terms of **E** and **B** fields) in presence of free charges and free current. [1]

(b) Convert the inhomogeneous Maxwell equations in terms of the magnetic vector potential **A** and the electromagnetic scalar potential Φ. [4]

(c) From your last result write down the inhomogeneous Maxwell equations in the Lorenz gauge and the Coulomb gauge. Clearly state the Lorenz gauge condition and the Coulomb gauge condition. [2]

(d) Given a scalar and vector potential satisfying Lorentz gauge condition,find the transformations of these scalar and vector potential such that Lorenz gauge condition is still satisfied. [3]

Question 8

List an experiment each that can be used for measuring the following fundamental constants in laboratory describing in brief (in two lines) the principle used in them. $[2 \times 5 = 10]$

(a) Electron charge (e)

(b) Boltzmann constant (k_B)

(c) Plank's constant (h)

(d) speed of light (c)

(e) permittivity of free space (ϵ_0)

Question 9

(a) Using IC-741, design a circuit which satisfies the following equation: [5]

$$V_{out} = \frac{dV_{in}}{dt} + 2V_{in} + 2$$

(b) For the following circuit, calculate the dc 'Q' point. [5]