LIGO-G1401144-v1

Interferometer configurations for Gravitational Wave Detectors

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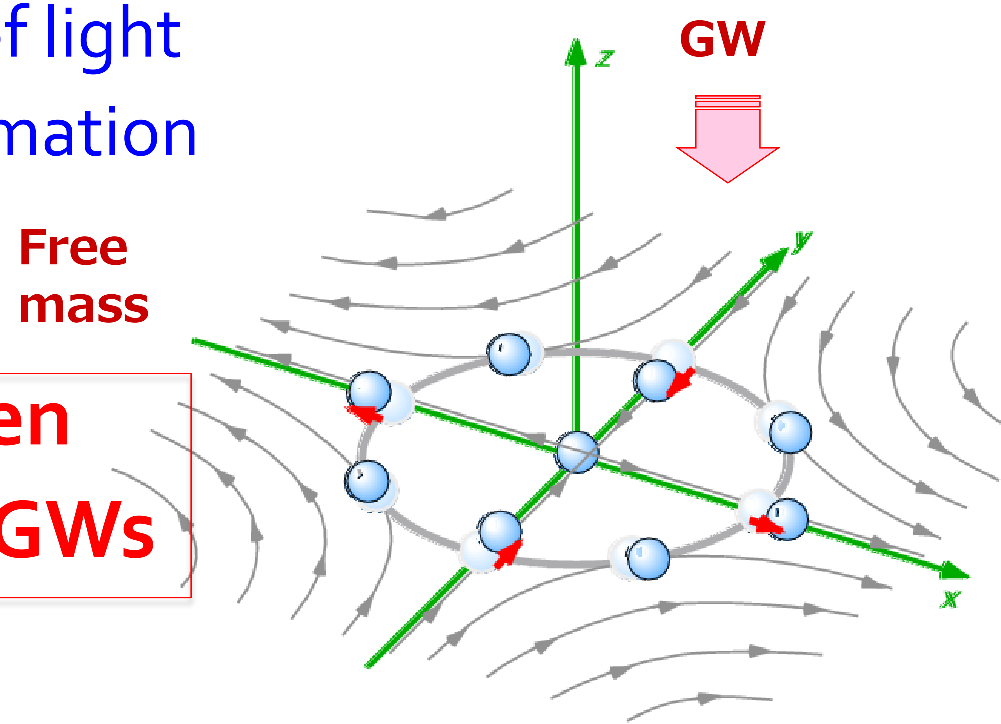
Gravitational wave

- **General Relativity**
  - Gravity = Spacetime curvature
  - Gravitational wave = Wave of spacetime curvature
- **Gravitational waves**
  - Generated by motion of massive objects
  - Propagates with speed of light
  - Cause quadrupole deformation of the spacetime

Measure strain between free masses to detect GWs



Introduction ~ Interferometer?

- Longer the baseline, the better
  - (displacement dx) = (Strain h) x (baseline L)
  - Let's use laser ranging for the measurement

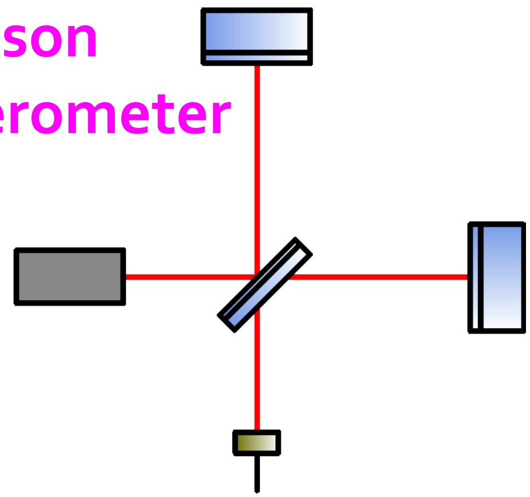
- e.g. Send pulses to measure the travel time?
  - Target: $h=10^{-23}$, $L=10$km => $dx = 10^{-19}$ m
  - Pulse timing resolution needs to be 3×10^{-28} s (!)
  - Current state-of-art pulse timing technology is 25 zepto s/rtHz (= 25×10^{-21} s/rtHz) Nature Photonics 7, 290-293 (2013).

- We need to measure phase of the laser light
  => use "laser interferometry"

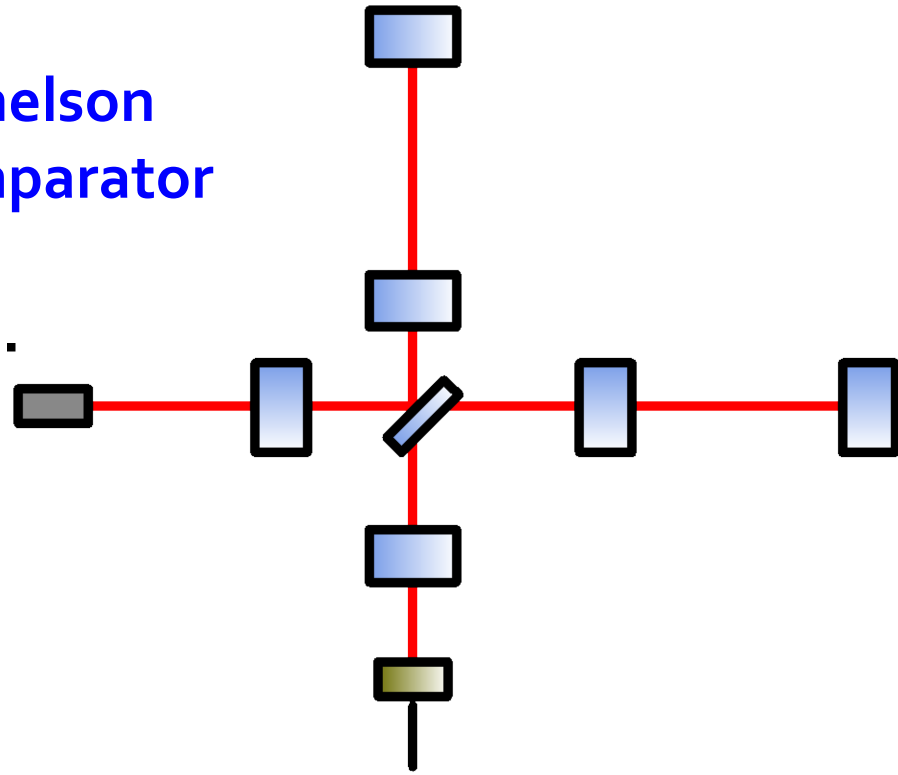
Introduction ~ Interferometer?

- Interferometry
  - Utilize characteristics of a Michelson interferometer as a length comparator

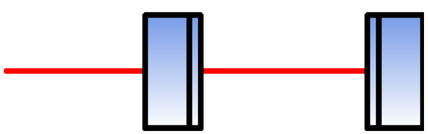
Michelson Interferometer



In reality…



Fabry-Perot cavity



Advanced LIGO
Dual-Recycled Fabry-Perot Michelson Interferometer

No worries: It's just a combination of MI and FPs

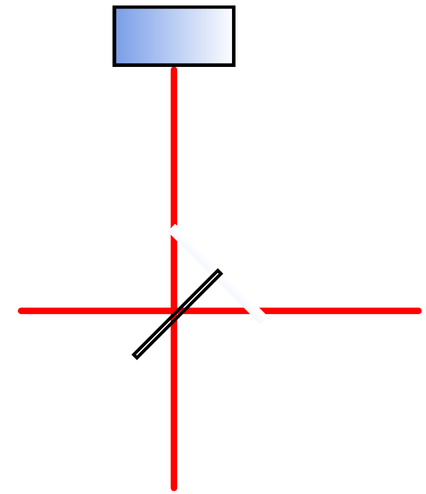
Michelson interferometer

- **Light intensity at the output port**
  - **Difference of the electric fields from the arms**

$$E_{\rm out} = \frac{1}{2}\left(e^{-i\phi_{\rm B}} - e^{-i\phi_{\rm A}}\right) E_{\rm in}$$

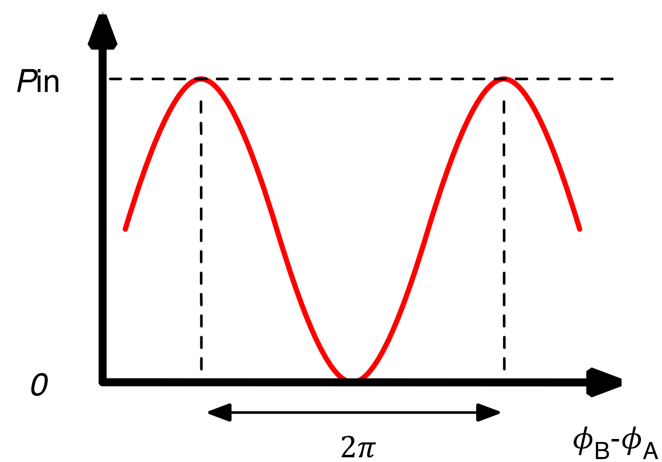
$$(\text{Roundtrip phase: } \phi_x = 4\pi\nu L_x/c)$$

$$E_{\rm out} = \left[i e^{-i(\phi_A+\phi_B)/2} \sin\frac{\phi_A - \phi_B}{2} \right] E_{\rm in}$$



$$P_{\rm out} = E_{\rm out} E^*_{\rm out} = \left(\sin^2 \frac{\phi_A - \phi_B}{2} \right) E_{\rm in}$$

$$= [1 - \cos(\phi_A - \phi_B)] \frac{P_{\rm in}}{2}$$



Output intensity is sensitive to the differential phase

Michelson interferometer

- **Frequency response of the Michelson to GWs**

$$\phi_A - \phi_B = \int_{t-2L/c}^{t} \Omega \left[1 + \frac{1}{2} h(t)\right] dt - \int_{t-2L/c}^{t} \Omega \left[1 - \frac{1}{2} h(t)\right] dt$$

$$= \int_{t-2L/c}^{t} \Omega h(t) dt$$

$$h(t) = h_0 e^{i\omega t}$$

Frequency response of the Michelson interferometer

$$\phi_A - \phi_B = \boxed{\frac{2L\Omega}{c} e^{-iL\omega/c} \frac{\sin(L\omega/c)}{L\omega/c}} \cdot h_0 e^{i\omega t}$$

$$= \frac{4\pi L}{\lambda_{\mathrm{opt}}} e^{-i2\pi L/\lambda_{\mathrm{GW}}} \frac{\sin(2\pi L/\lambda_{\mathrm{GW}})}{2\pi L/\lambda_{\mathrm{GW}}} \cdot h_0 e^{i\omega t}$$

Ω: optical angular frequency, λ_{OPT} laser wavelength
ω: angular frequency of GW, λ_{GW} wavelength of GW

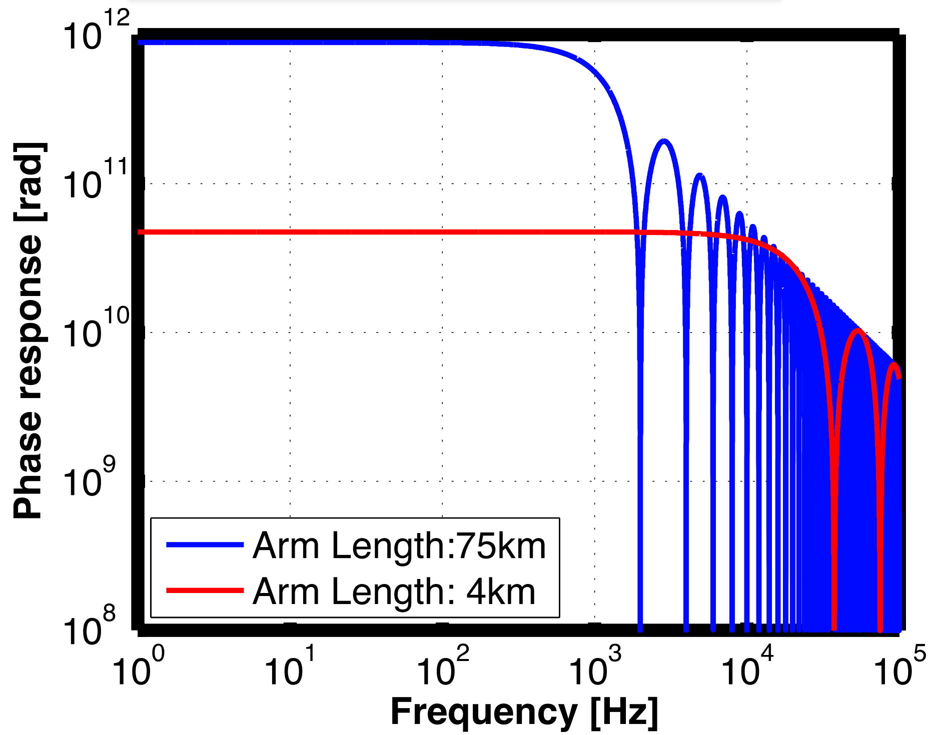
Jean-Yves Vinet, et al
Phys. Rev. D 38, 433 (1988)

Michelson interferometer

- **Frequency response of the Michelson to GWs**

$$\Delta\phi = \boxed{\frac{2L\Omega}{c} e^{-iL\omega/c} \frac{\sin(L\omega/c)}{L\omega/c}} \cdot h_0 e^{i\omega t}$$

DC Response
longer ->
larger



Cut off freq
longer -> lower

Notch freq
f = n c / (2 L)
(Notches occur
when L = n λ/2)

Michelson arm length optimized for 1kHz GW
-> 75km, too long!

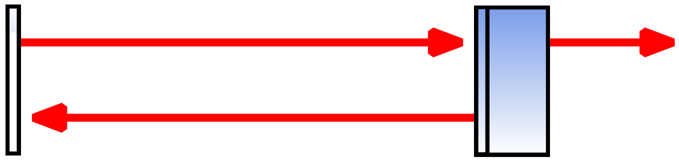
Fabry-Perot optical resonator

- **Storing light in an optical cavity**
  - Field equations

$$\boxed{E_{\mathrm{cav}} = t_1 E_{\mathrm{in}} + r_2 e^{-i\phi} E_{\mathrm{cav}}}$$

$$E_{\mathrm{t}} = t_2 e^{-i\phi/2} E_{\mathrm{cav}}$$

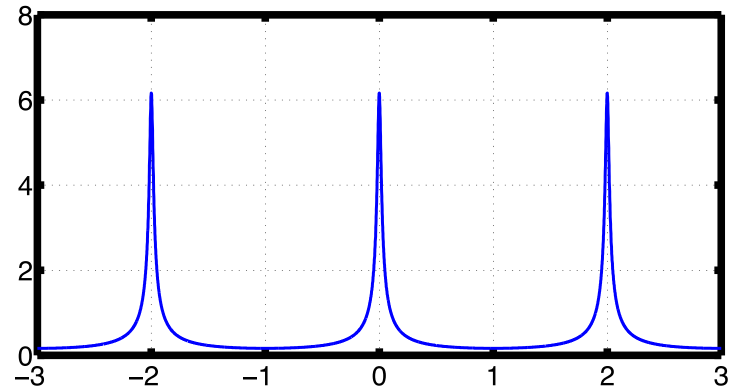
$$E_{\mathrm{ref}} = -r - 1 + t_1 r_2 e^{-i\phi} E_{\mathrm{cav}}$$



$$\boxed{\frac{E_{\mathrm{cav}}}{E_{\mathrm{in}}} = \frac{t_1}{1 - r_1 r_2 e^{-i\phi}}}$$

$$\frac{E_{\mathrm{r}}}{E_{\mathrm{in}}} = -r_1 + \frac{t_1^2 r_2 e^{-i\phi}}{1 - r_1 r_2 e^{-i\phi}}$$

$$\frac{E_{\mathrm{t}}}{E_{\mathrm{in}}} = \frac{t_1 t_2 e^{-i\phi/2}}{1 - r_1 r_2 e^{-i\phi}}$$



ad]

Very fast phase response

Fabry-Perot optical resonator

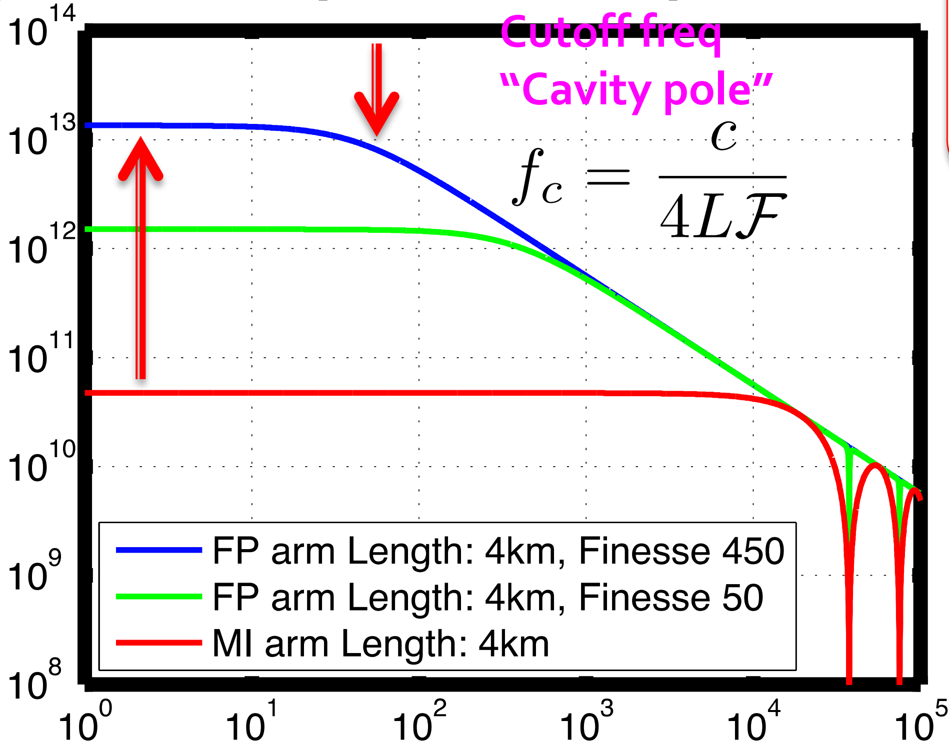
- **Storing light in an optical cavity**

Finesse

$$\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

DC Response amplification

$$N = 2\mathcal{F}/\pi$$

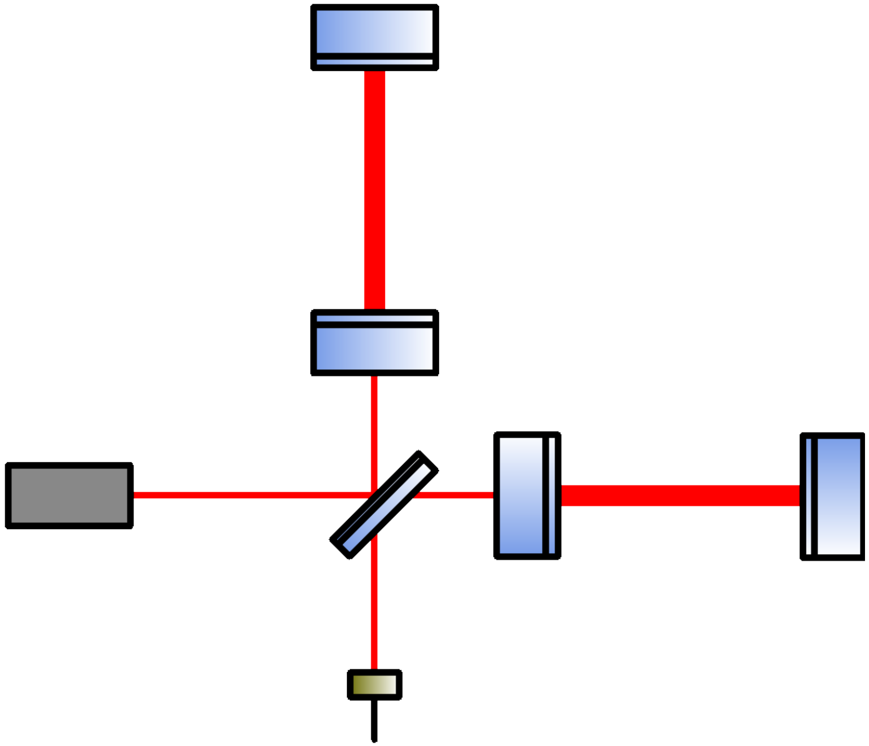


1. **FP increases stored power in the arm**
2. **FP increases accumulation time of the signal**

=> Above the roll-off, increasing F does not improve the response

Fabry-Perot Michelson Interferometer

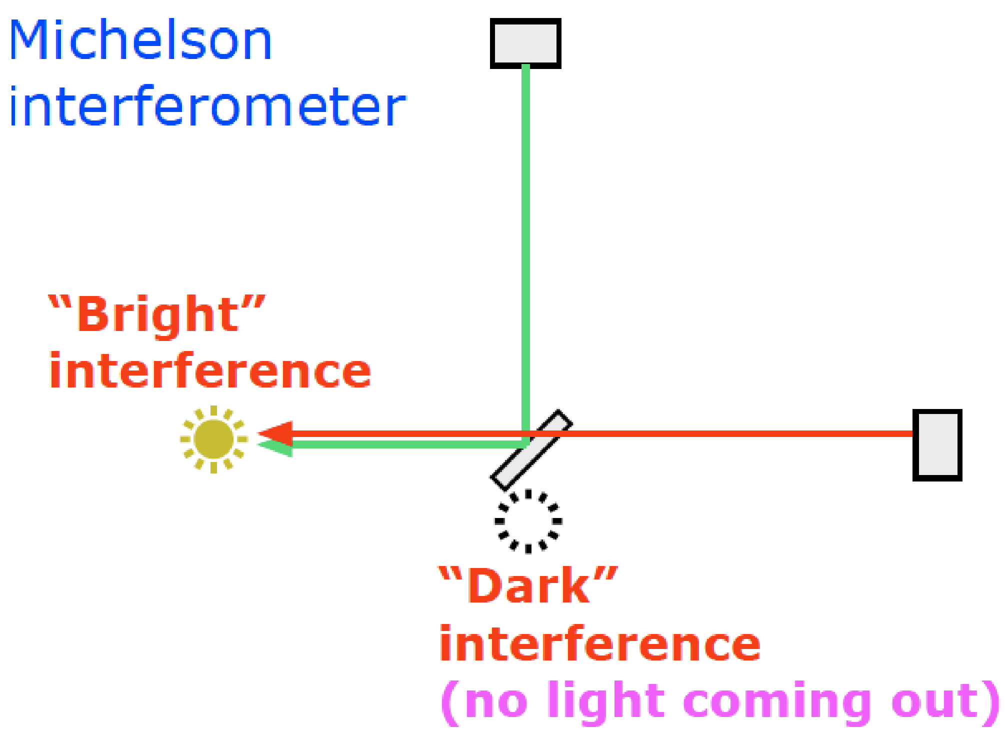
- Differential nature of the Michelson
  + Longer photon storage time of Fabry-Perot cavities
  = Fabry-Perot Michelson Interferometer



Basic form of the modern interferometer GW detector

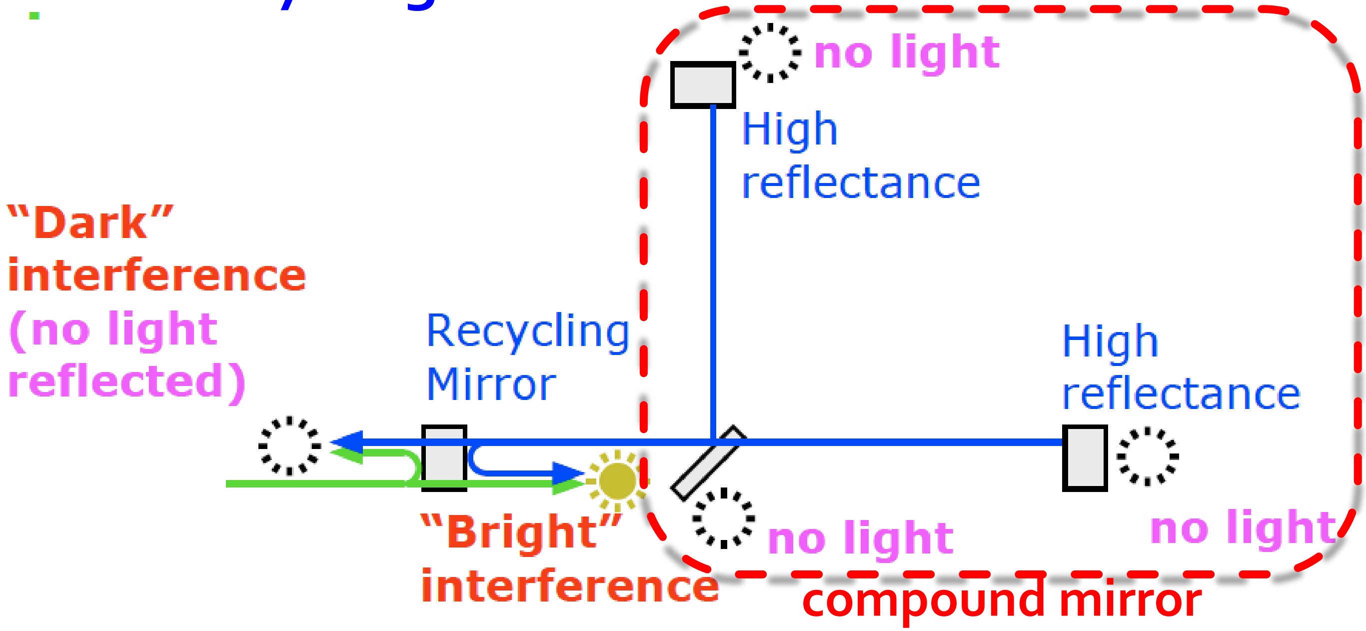
Optical Recycling Technique

- Power recycling
  - When the Michelson interferometer is operated at a "dark fringe", most of the light goes back to the laser side



Optical Recycling Technique

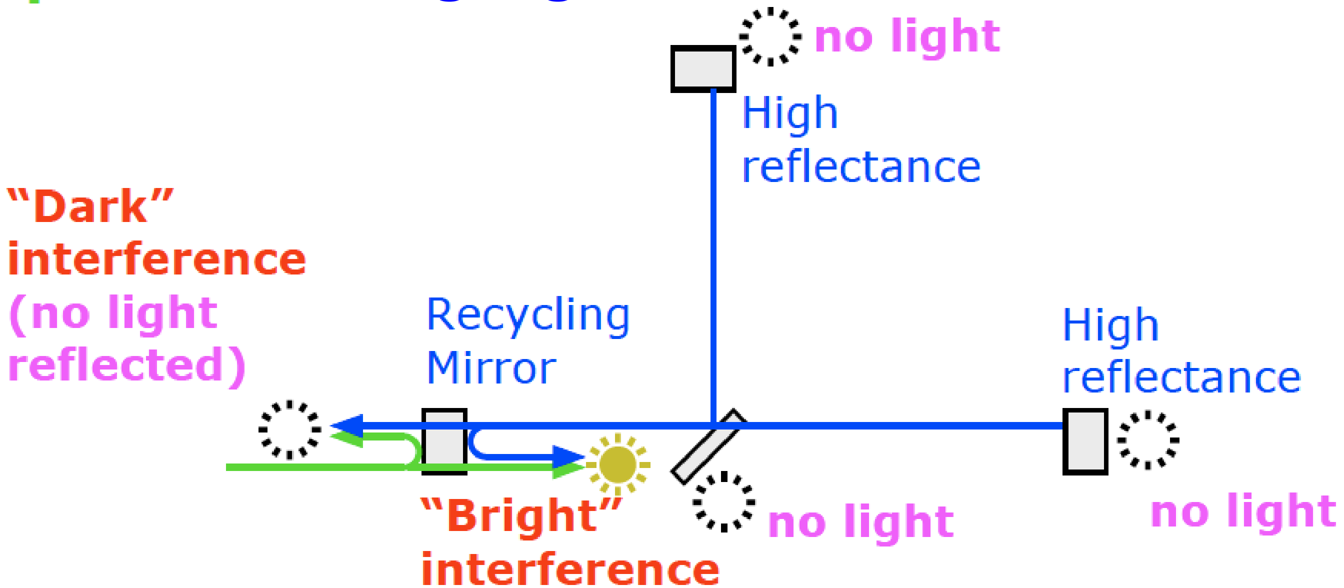
- Power recycling
  - Let's reuse the reflected light
  - Place a mirror in front of the interferometer
    to form a cavity with the Michelson (compound mirror)
    "Power Recycling Mirror"



  - The internal light power is increased
    = equivalent to the increase of the input laser power

Optical Recycling Technique

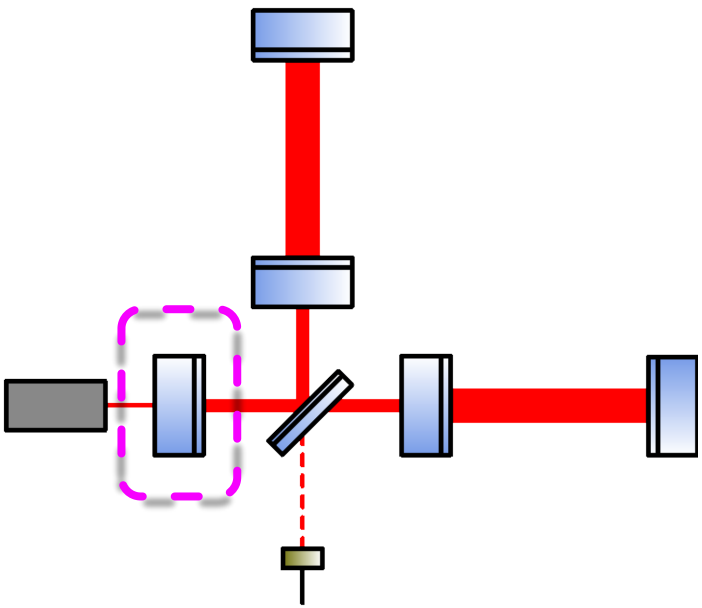
- Power recycling
  - BTW, all the output ports are made dark. Where does the light go?



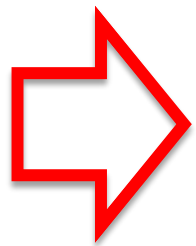
  - In the ideal power recycling, all input power is internally consumed via optical losses (absorption & scattering)

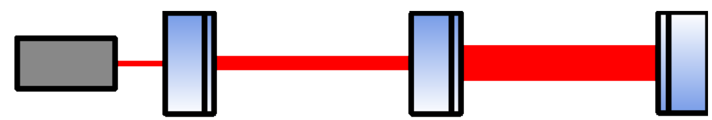
Optical Recycling Technique

- Power-recycled Fabry-Perot Michelson Interferometer
  - Internal light power in the arms is increased

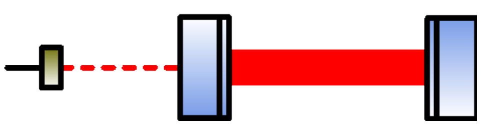


From the laser side /
common arm length change
It looks like a three mirror cavity
= high finesse cavity



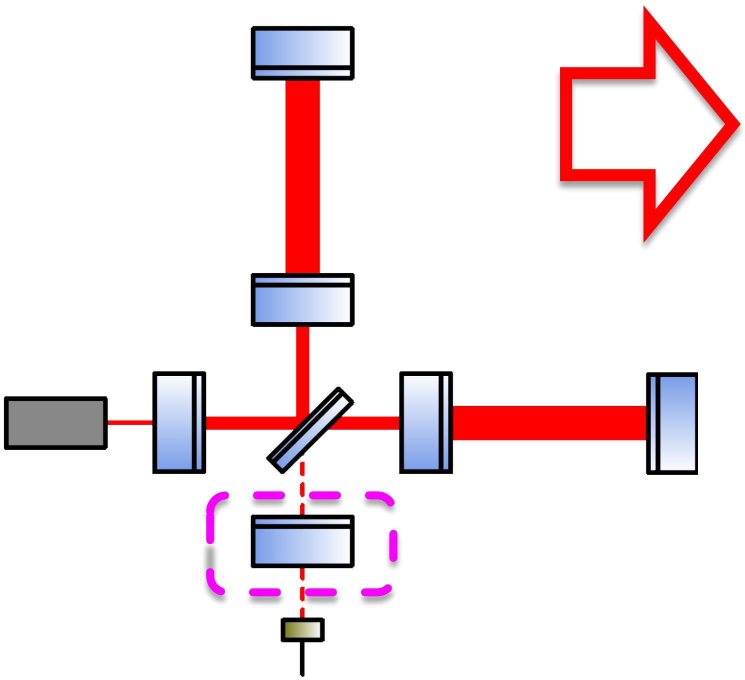


For the differential motion (=GW)
It looks like just an arm cavity

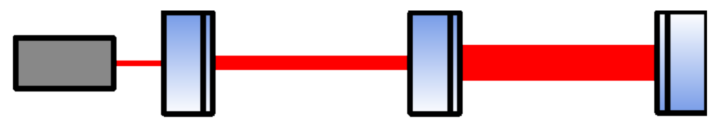


Optical Recycling Technique

- Dual-recycled Fabry-Perot Michelson Interferometer
  - Another mirror is added at the dark port
    "Signal Recycling Mirror"
  - Dual recycling allows us to set different storage times for common and differential modes



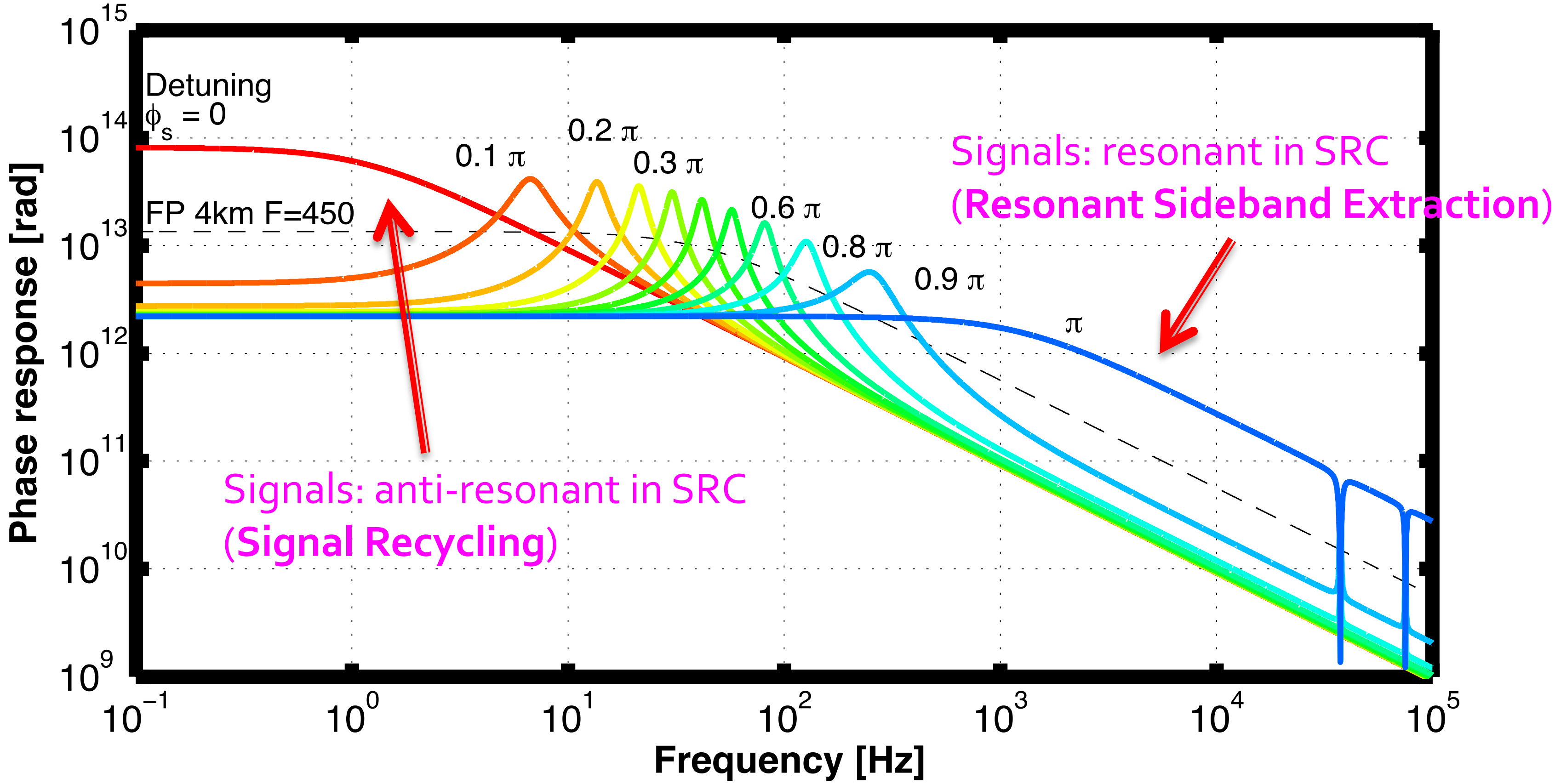
Common mode
= high finesse three mirror cavity



Differential mode (=GW)
= low (or high) finesse three mirror cavity

To tuned or not to tune

- Bandwidth of the detector can be changed
  - by changing the resonant phase of the signal recycling cavity (SRC)



- Optimize the curve depending on the noise shape
- Dynamic signal tracking

Summary

- Optical phase measurement => Interferometry

- Michelson interferometer: requires too long arm
- Fabry-Perot arm: longer light storage time

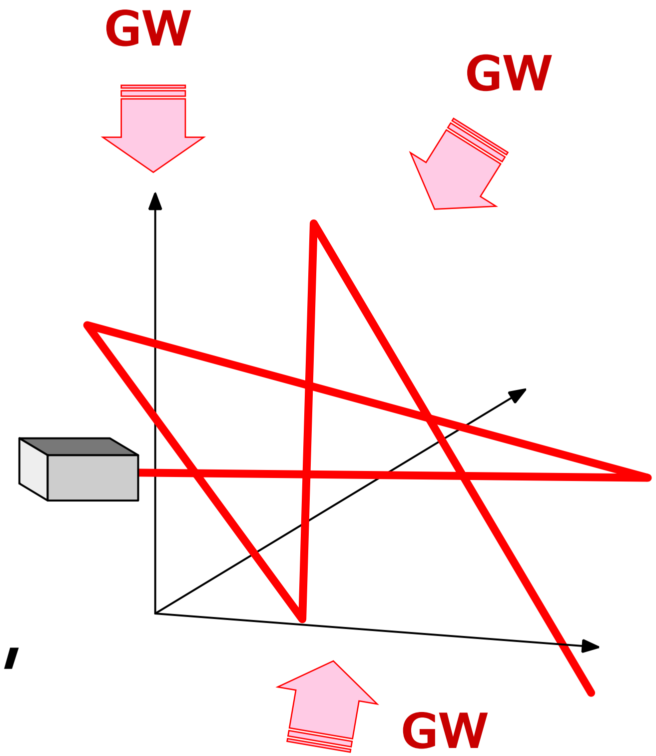
- Optical recycling technique:
  allows us to set different storage time for the incident light and the GW signals
  - Power recycling: maximize the stored light power
  - Resonant sideband extraction: optimize the signal bandwidth

Advanced topics

- **Angular & frequency response of an interferometer**
  - Up to this point GWs from the zenith was assumed.
  - What is the response to GWs with an arbitrary angle?
  - What is the frequency response of the detector for such GWs?

  - Draw an arbitrary optical path.
    What is the angular and frequency response of such a path?

  - Can we use numerical "optimization" for certain criteria?
    e.g.
    better sky coverage, directive beaming, for certain source frequency, etc...



Advanced topics

- **Angular & frequency response of aninterferometer**
  - R. Schilling, Class. Quantum Grav. 14 (1997) 1513-1519

2.1. Single round trip

We will assume a gravitational wave propagating along the Z direction, with its polarization axes being parallel to the X/Y axes. In the simplest case the arm lies entirely in the X–Y plane, but in general there will be a tilt angle ϑ between the direction of the arm and the X–Y plane. With the single pass of a light beam travelling along the arm and measuring its length ℓ we find

$$\ell(t) = \ell_0 + \tfrac{1}{2} c \cos^2\vartheta \int_{t_0-\ell_0/c}^{t_0} h\left[t + t'(1-\sin\vartheta)\right] \mathrm{d}t'. \tag{1}$$

For a sinusoidal gravitational wave $h(t) = \hat{h}\exp(\mathrm{i}\omega t)$ and $t_0 = 0$ this becomes

$$\ell(t) = \ell_0 + \tfrac{1}{2}\hat{h}\ell_0 \cos^2\vartheta \,\mathrm{sinc}\left[\frac{\omega\ell_0}{2c}(1-\sin\vartheta)\right] \exp\left[\mathrm{i}\omega t - \mathrm{i}\frac{\omega\ell_0}{2c}(1-\sin\vartheta)\right], \tag{2}$$

where the sinc function is defined as $(\sin x)/x$. A complete round trip consists of the concatination of a forward and a return pass; for the latter we have to replace ϑ by $-\vartheta$, and we have to fulfil a continuation condition for the phase of the induced signal at the return point (mirror or transponder). For the time-varying part of ℓ this leads to

$$\begin{aligned}\delta\ell(t) = \tfrac{1}{2}\hat{h}\ell_0\cos^2\vartheta\{&\mathrm{sinc}[\pi\Omega(1-\sin\vartheta)]\exp[-\mathrm{i}\pi\Omega(3+\sin\vartheta)] \\ &+ \mathrm{sinc}[\pi\Omega(1+\sin\vartheta)]\exp[-\mathrm{i}\pi\Omega(1+\sin\vartheta)]\}\exp(\mathrm{i}\omega t),\end{aligned} \tag{3}$$

where we have introduced a normalized frequency Ω with $2\pi\Omega = \omega\ell_0/c$. The result of equation (3) can also be expressed in the form of a *normalized* antenna transfer function $\mathcal{T} = 2\delta\ell(t)/[\ell_0\hat{h}\exp(\mathrm{i}\omega t)]$ as

$$\begin{aligned}\mathcal{T} = \cos^2\vartheta\{&\mathrm{sinc}[\pi\Omega(1-\sin\vartheta)]\exp[-\mathrm{i}\pi\Omega(3+\sin\vartheta)] \\ &+ \mathrm{sinc}[\pi\Omega(1+\sin\vartheta)]\exp[-\mathrm{i}\pi\Omega(1+\sin\vartheta)]\}.\end{aligned} \tag{4}$$

Figure 1(a) shows the magnitude of the normalized one-arm transfer function $\mathcal{T}_1$ for a single round trip and $\vartheta = 0^\circ$, indicated separately for the forward pass, the return pass and the full round trip. In the case shown, the transfer functions for the forward and return pass are identical in magnitude, only differing in phase, which leads to the additional zeros in the full round-trip response at frequencies $\Omega = \frac{1}{2}(2k-1)$.

The response for a tilt of $\vartheta = 45^\circ$ is shown in figure 1(b), revealing two interesting facts: the zeros of the round-trip response have moved up to much higher frequencies, from multiples of $\Omega = \frac{1}{2}$ to ones of $\Omega = 3.41$, and the transfer function can take values that are even above the envelope for $\vartheta = 0^\circ$. It turns out that the well known response for the tilt $\vartheta = 0^\circ$ is, in fact, the exception rather than the normal case, since most of the zeros (caused by cancellation) appear at normal incidence only.

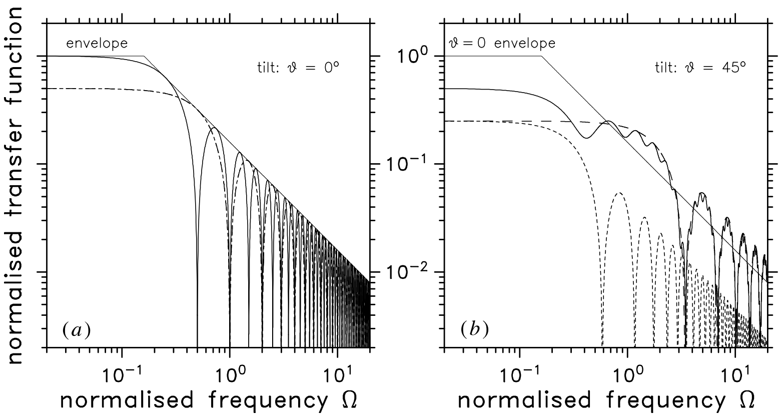


Figure 1. Magnitude of the normalized transfer function for a single round trip in a single arm and a tilt of (a) 0° and (b) 45°. Full curve, round trip; long broken curve, forward pass; short broken curve, return pass.