LIGO-G1401145-v1

Noises in Gravitational Wave Detectors

Koji Arai / Stan Whitcomb – LIGO Laboratory / Caltech

Introduction ~ Noise?

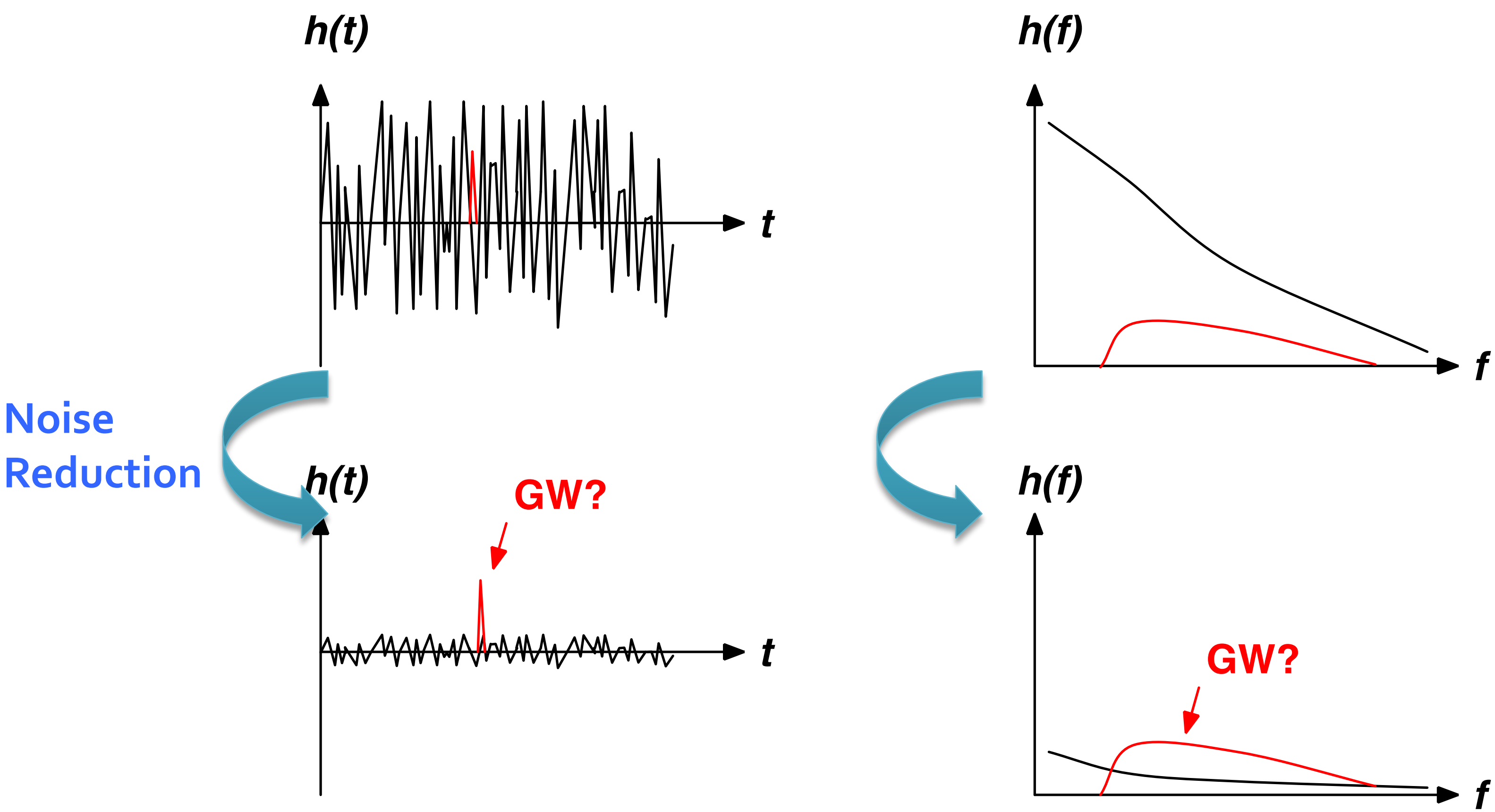
- **GW detection**
  Data stream of differential arm strain

- **Once recorded:**
  Signals and noises are indistinguishable
  What we can do is to catch “likely” features

- **Reduce any kind of noises!**

Introduction ~ Noise?

- Time domain vs frequency domain



- Time domain: transient noises
  Frequency domain: stationary noises

Introduction ~ Noise?

- **Power Spectral Density (PSD)**
  Double sided PSD (-Infinity < f < Infinity)

$$S_{\mathrm{DS}}(f) = \lim_{T\to\infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} x(t) e^{-2\pi i f t} \mathrm{d}t \right|^2$$

- **Single sided PSD (0 <= f < Infinity)**

$$S_x(f) = 2S_{\mathrm{DS}}(f)$$ [x_{unit}^2 / Hz]

- **Linearized PSD:**

$$G_x(f) = \sqrt{S_x(f)}$$ [x_{unit}/ sqrtHz]

Introduction ~ Noise?

- **Parseval's Theorem** for signal RMS and PSD

$$\overline{x^2(t)} = \int_0^\infty S_x(f)df$$
$$\equiv x_{\mathrm{RMS}}^2$$

Root Mean of *x*(*t*):

average signal power density (per sec)
(cf. variance, std deviation)

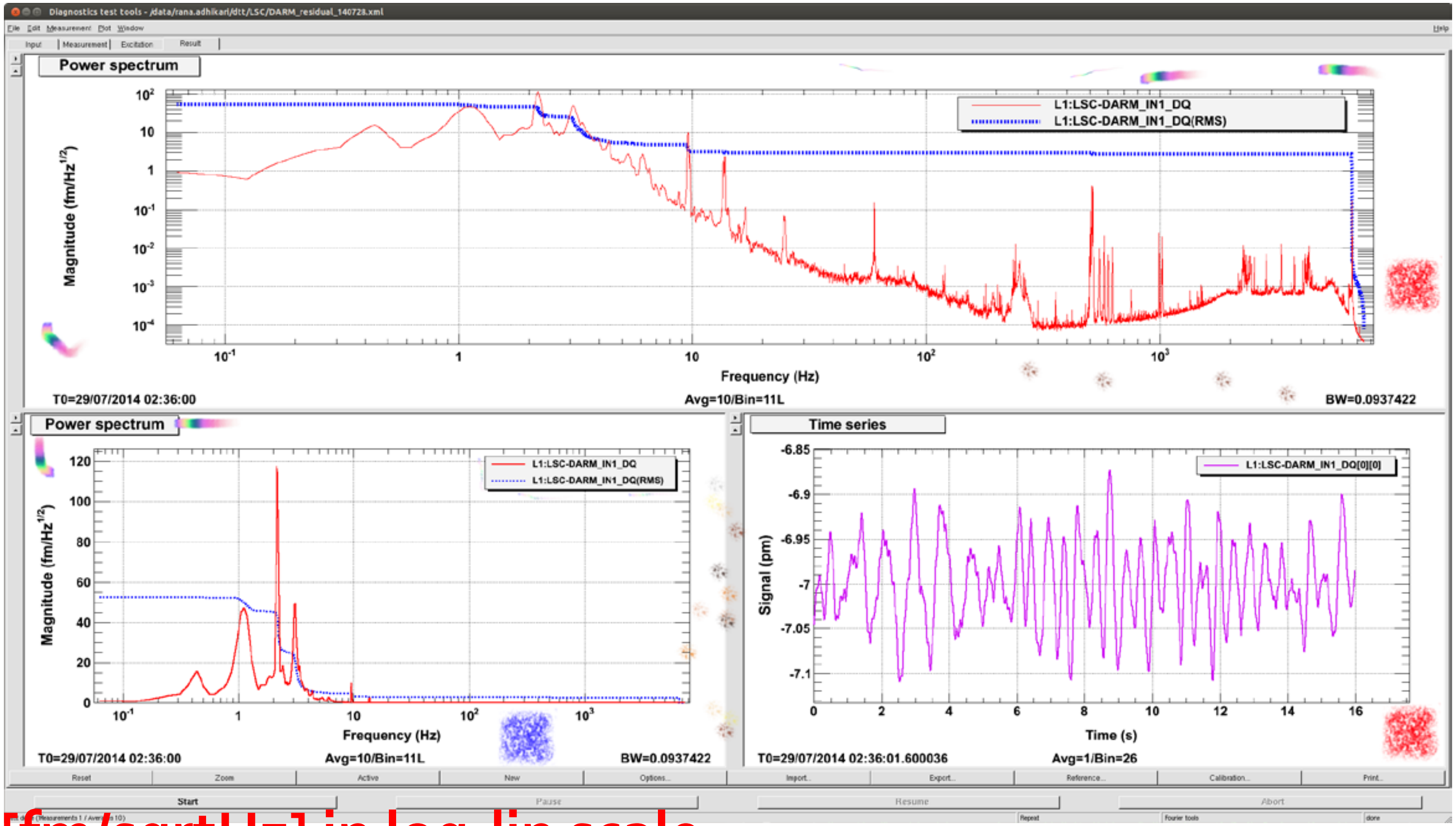
PSD *Sx*(*f*):

power density per frequency (per sec)

Introduction ~ Noise?

Example

PSD [fm/sqrtHz] in log-log scale, RMS [fm] ~ 50fm = 0.05pm



PSD [fm/sqrtHz] in log-lin scale

RMS [fm]

Time series [pm]

Components of the interferometer

- 3 fundamentals of the GW detector
- Mechanics
- Optics
- Electronics

Components of the interferometer

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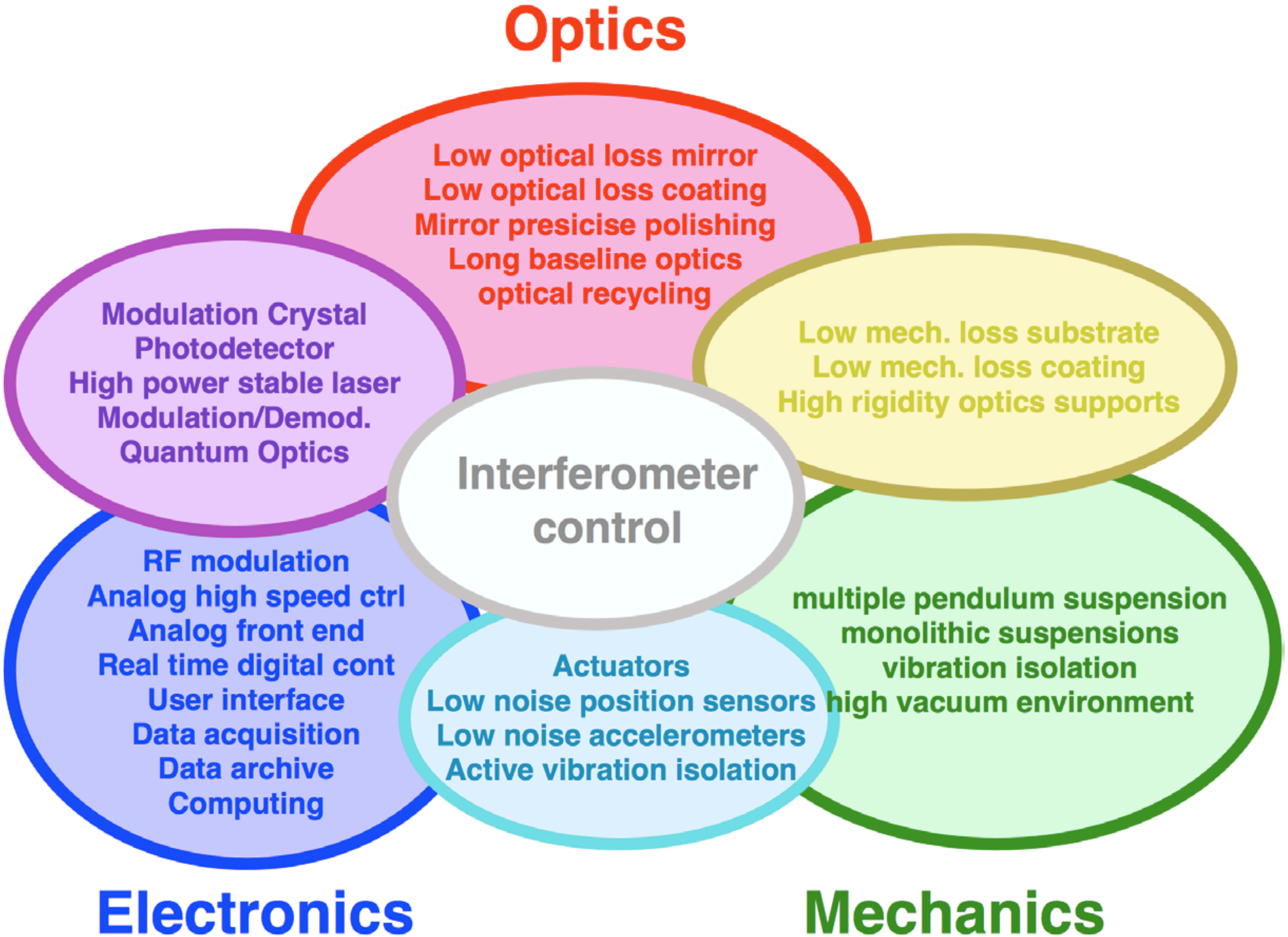
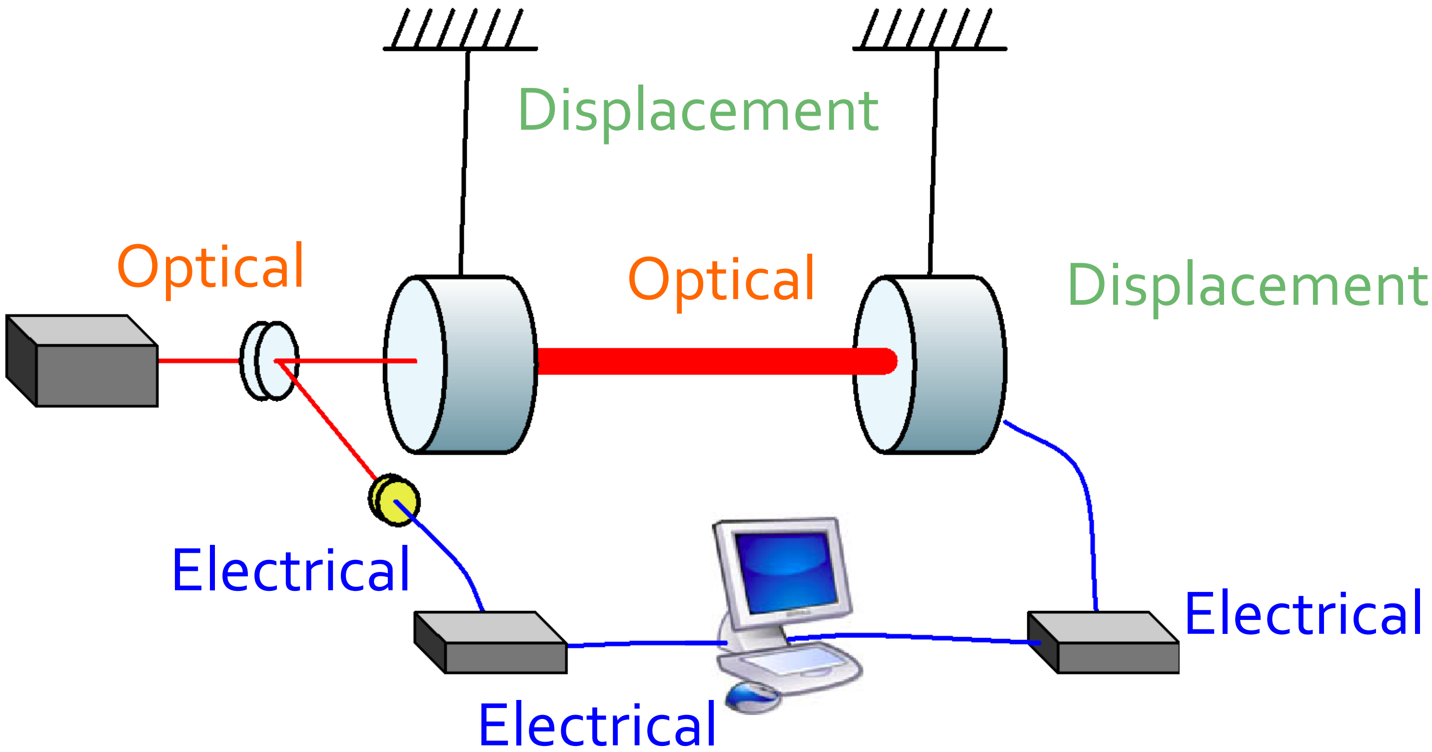
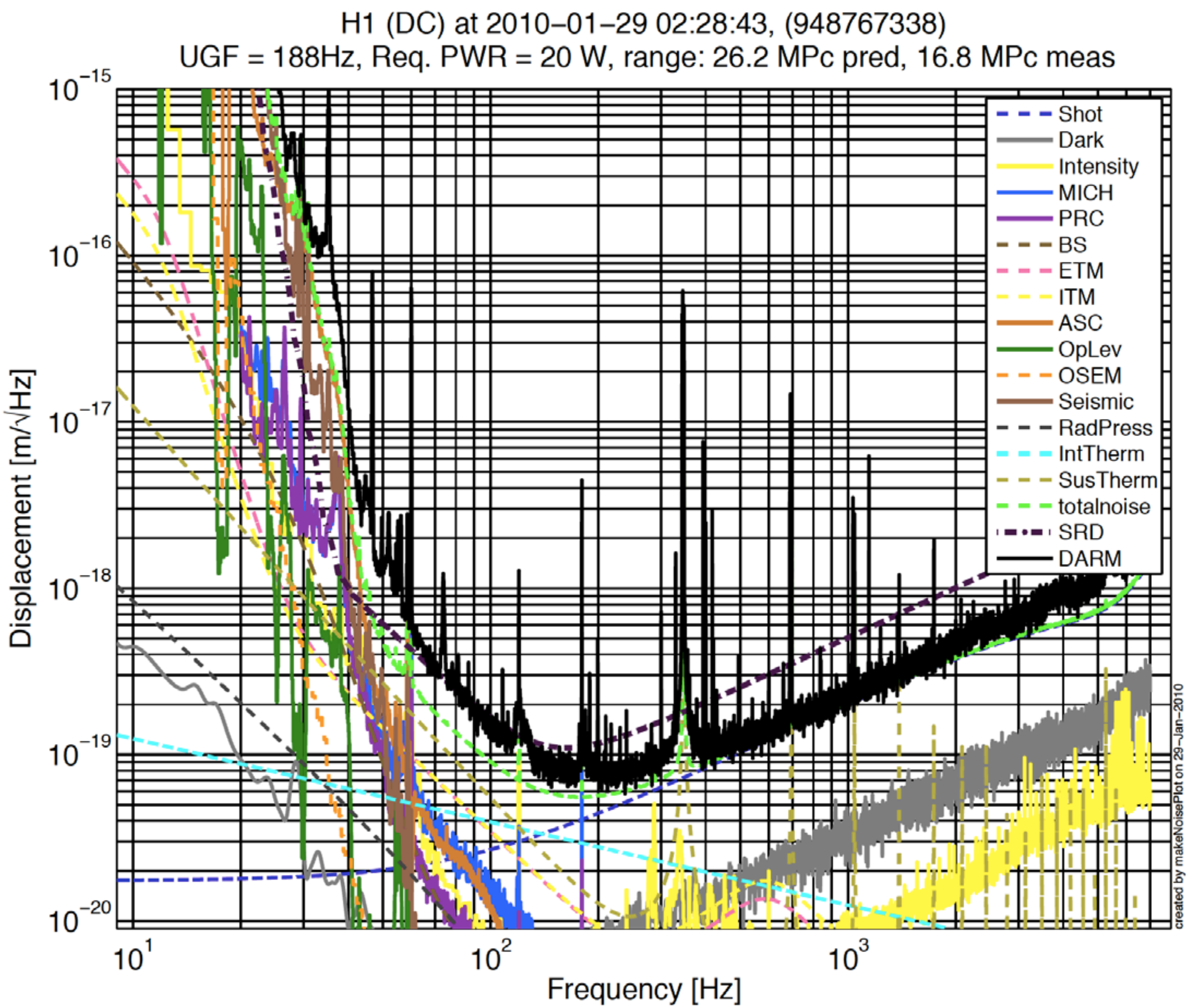


Noise categories

- 3 fundamentals of the GW detector
- Mechanics -> Displacement noises
- Optics -> Optical noises
- Electronics -> Electrical noises



Noise budget



Laser shot noise
Laser radiation pressure noise
thermal noise
seismic noise
Laser intensity /frequency noise

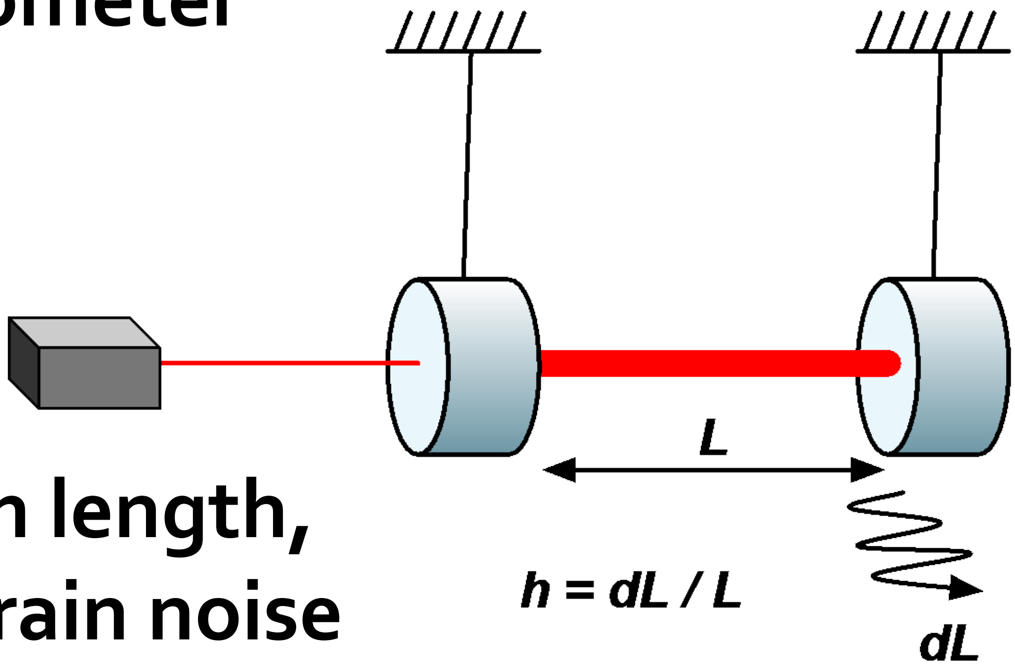
electronics noise
digitization noise
angular control noise

……

Displacement noises

Displacement noise

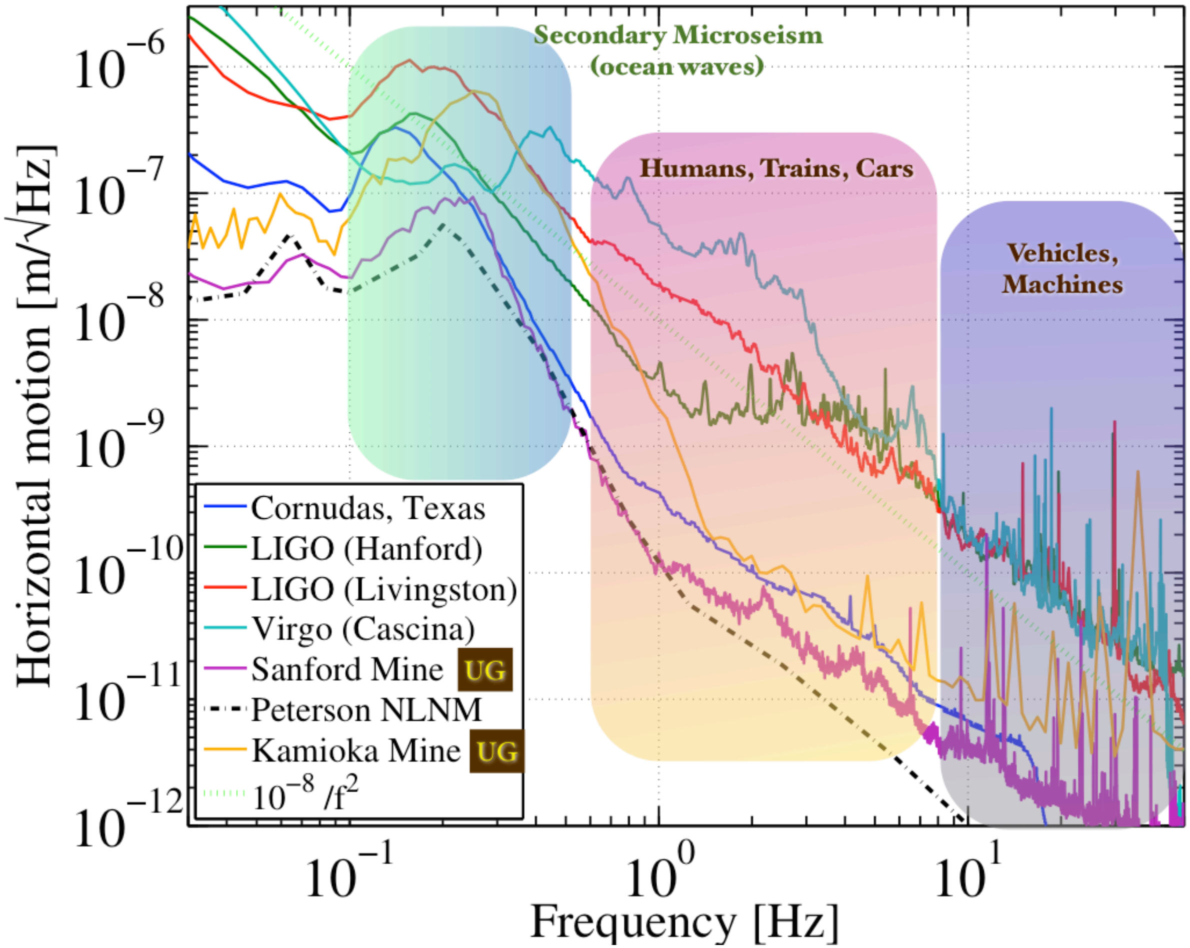
- Mechanical displacement sensed by a laser interferometer



- The longer the arm length, the smaller the strain noise
  - Seismic noise
  - Thermal noise
  - Newtonian Gravity noise

Displacement noise

- **Seismic noise**
  - Even when there is no noticeable earth quake…

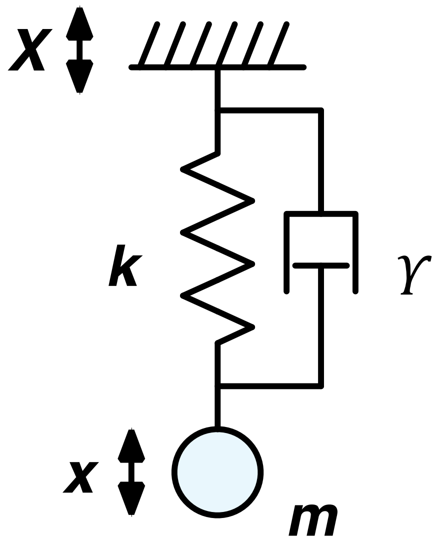


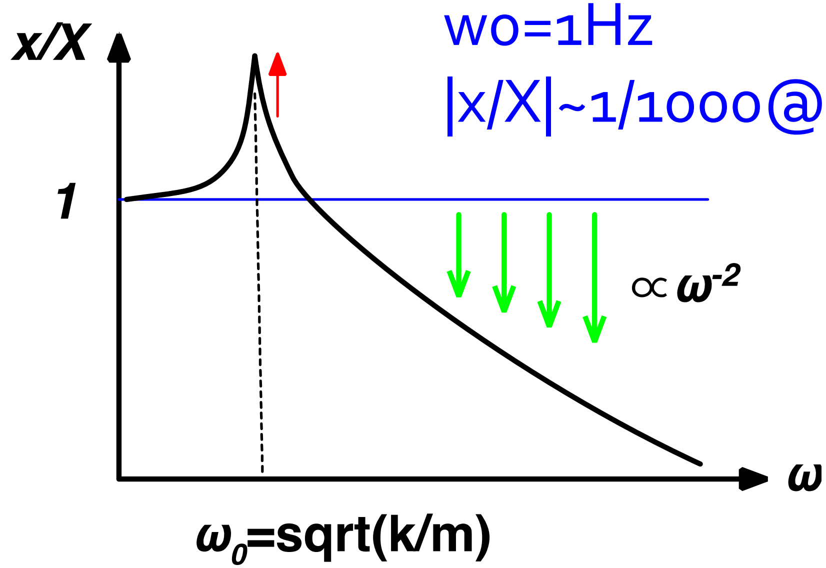
Target disp. noise 10^{-20}m/rtHz

http://link.aps.org/doi/10.1103/RevModPhys.86.121 (http://arxiv.org/abs/1305.5188)

Displacement noise

- **Vibration isolation ~ utilize a harmonic oscillator**
  - A harmonic oscillator provides vibration isolation above its resonant frequency





$$m\ddot{x} = -k(x - X) - \gamma(\dot{x} - \dot{X})$$

$$\left(\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2\right)\tilde{x} = \left(\omega_0^2 + i\frac{\gamma}{m}\omega\right)\tilde{X}$$

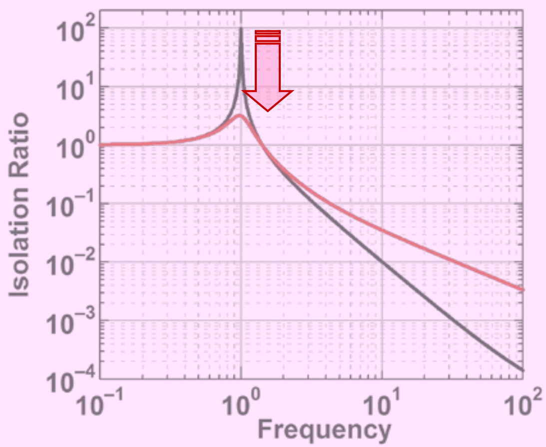
$$\frac{\tilde{x}}{\tilde{X}} = \frac{\omega_0^2 + i\frac{\gamma}{m}\omega}{\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2}$$

Displacement noise

- How to get more isolation?

Damping

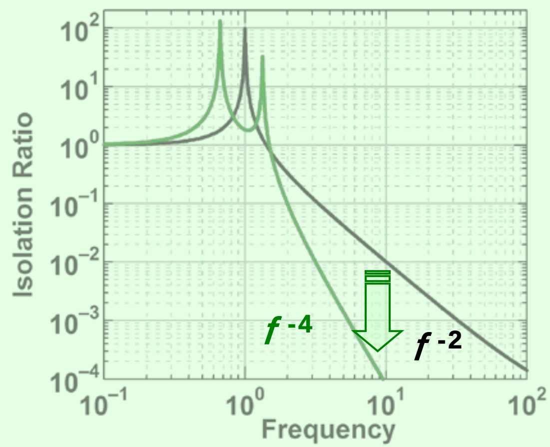
Lower the peak height



Worse isolation

Multi stage

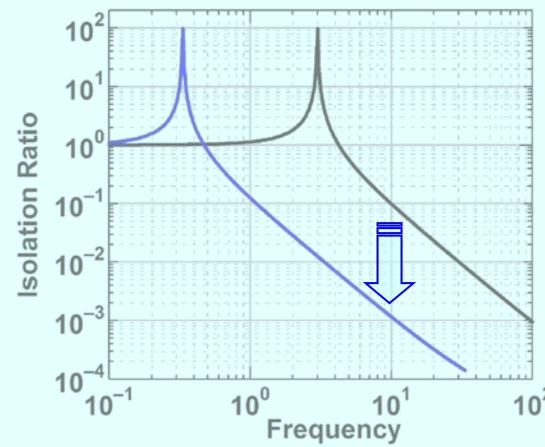
Steeper isolation curve



More peaks

Lower resonant freq

Better isolation

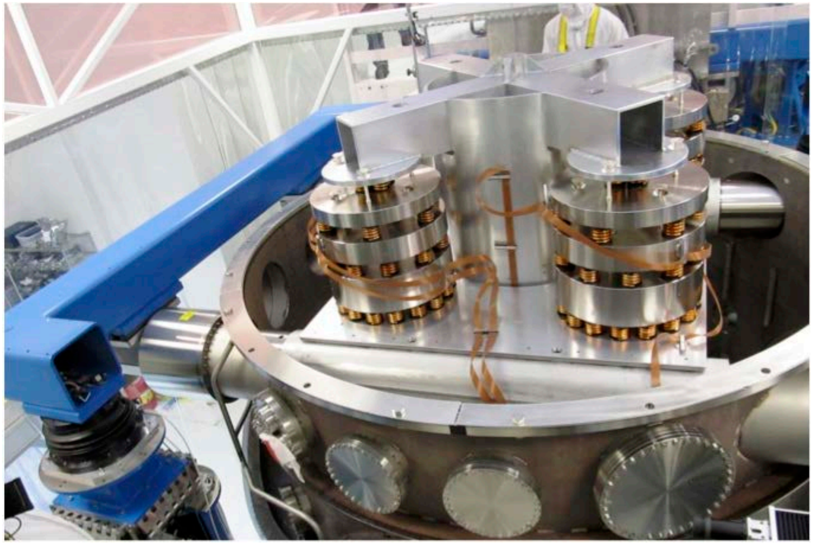


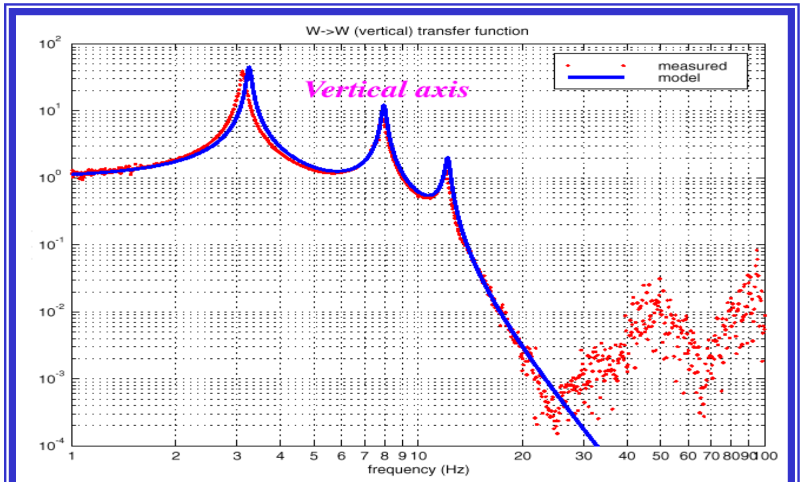
Complex to realize

- In practice: employ combination of these measures

Displacement noise

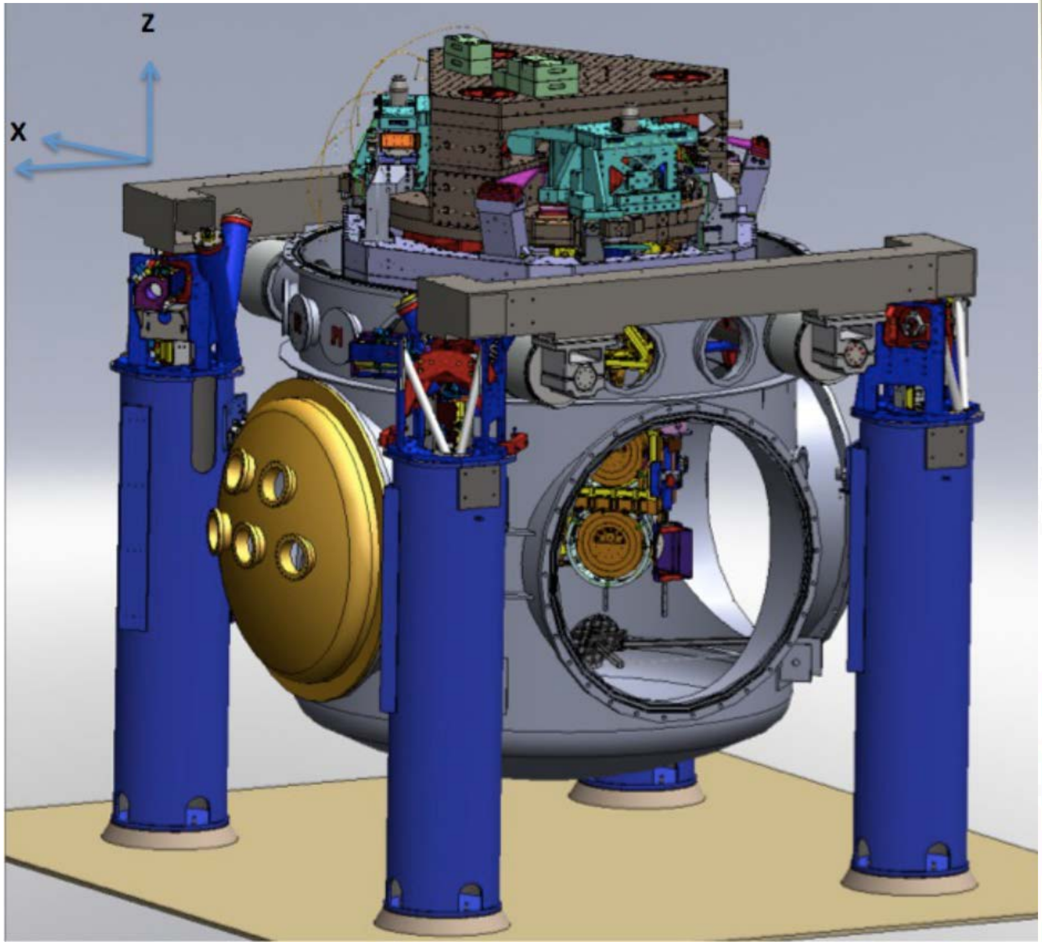
- **Initial LIGO vibration isolation**
- **Hydraulic active isolation / Isolation stack / Single Pendulum**

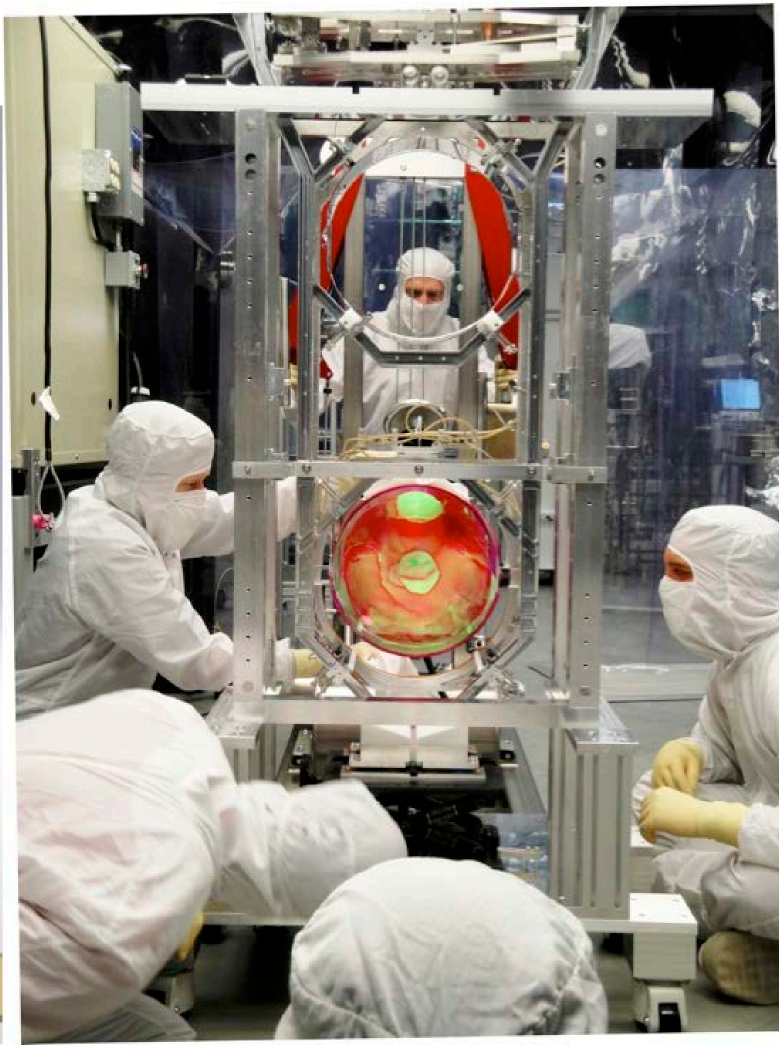




Displacement noise

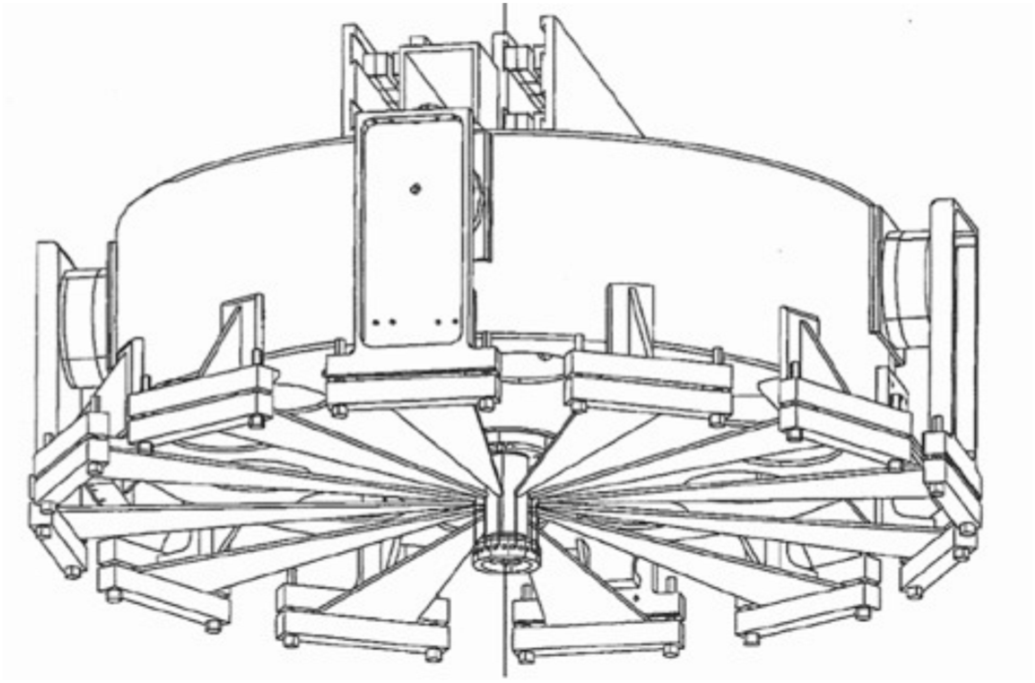
- **Advanced LIGO vibration isolation**
- **Hydraulic active isolation / In-vacuum Active Isolation Platforms / Multiple Pendulum**

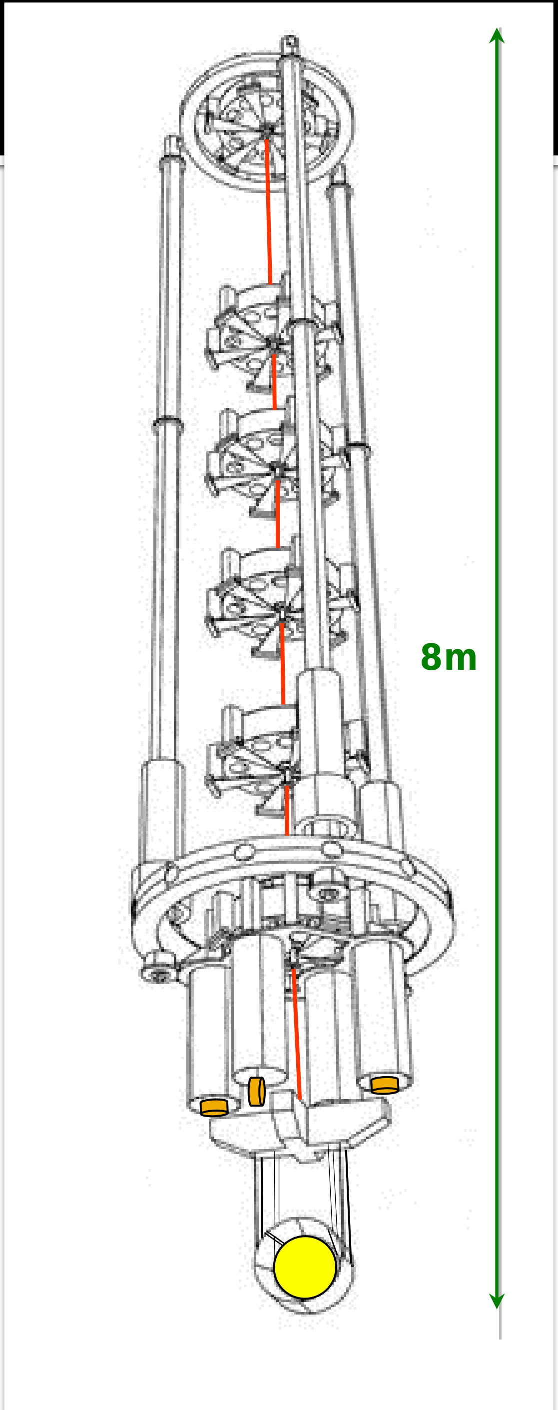




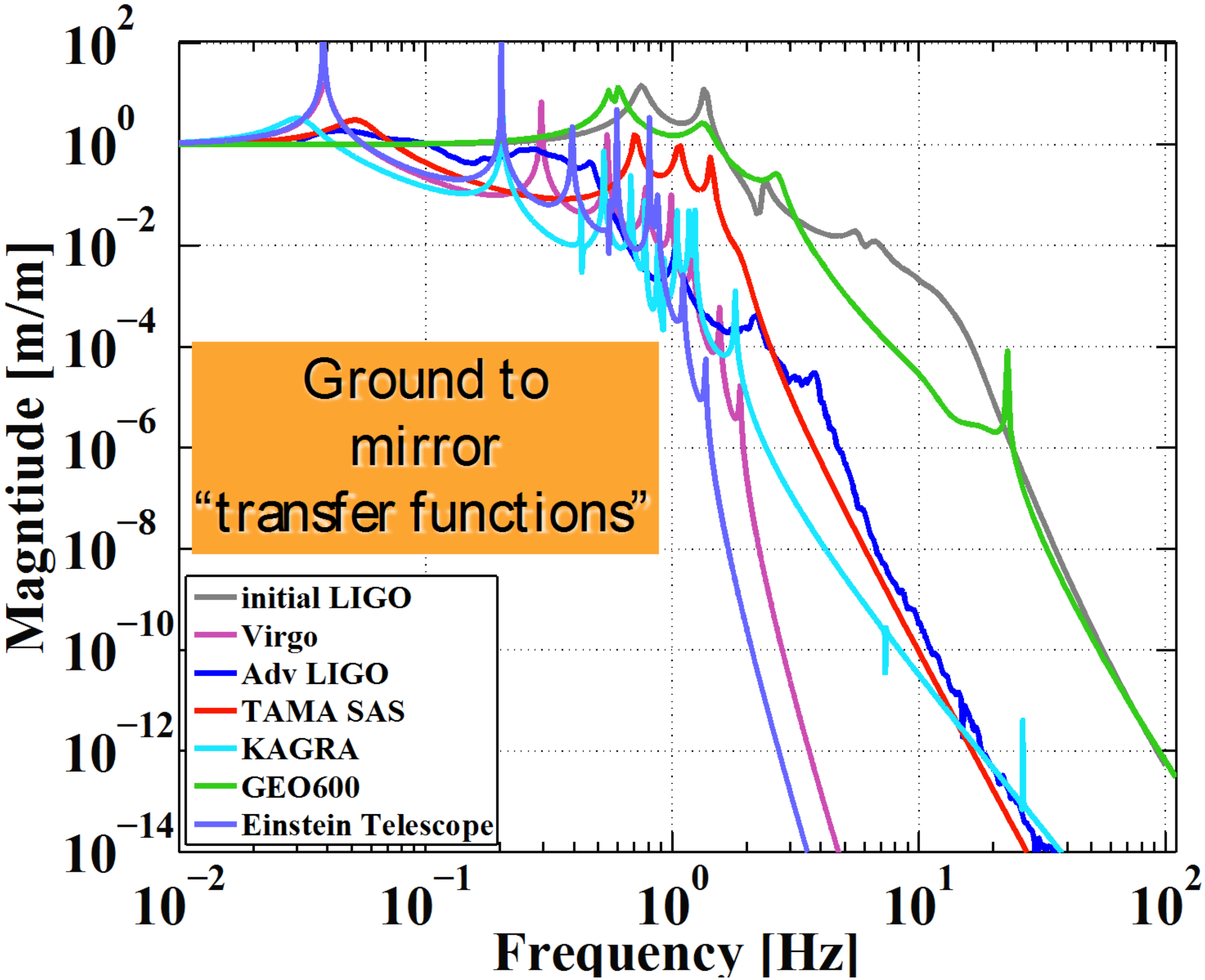
Displacement noise

- Virgo: super attenuator
  - 8m high
  - 9 stages in horizontal
  - 6 stages in vertical





Displacement noise

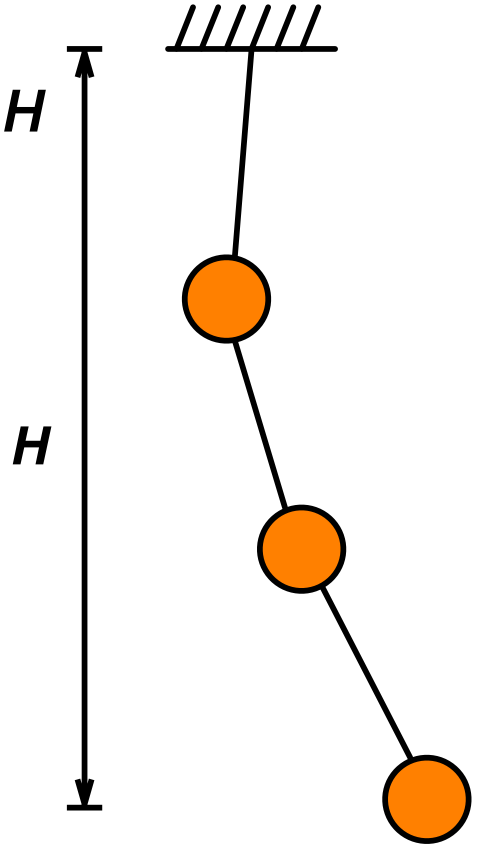


http://link.aps.org/doi/10.1103/RevModPhys.86.121 (http://arxiv.org/abs/1305.5188)

Displacement noise

Question:

- ***n***-stage multiple pendulum with fixed height of ***H***
- How many stages ***n*** do we need to realize the vibration isolation of ***A*** at frequency of ***f***?
- For a given ***A*** what is the minimum ***f***, we can realize by increasing ***n***?



(Mass distribution)

- For equal m for each stage
  or
- For arbitrary mass m_i and length h_i

Displacement noise

- Thermal noise:
- System in thermal equilibrium
  - the system can dissipate its energy to the heat bath
  - the system is thermally excited by thermal fluctuation

- Mechanical thermal noises
  - suspension thermal noise
  - mirror substrate thermal noise
  - mirror coating thermal noise

Displacement noise

- Fluctuation Dissipation Theorem
- Friction: interaction with "bath" = huge number of degrees of freedom
- Fluctuation force: produced by huge number of d.o.f.
- Dissipation and fluctuation have certain relationship

    system description (Langevin equation)

$$m\ddot{q} + R\dot{q} = \mathcal{F} + F'(t)$$

q: generalized coordinate m: generalized mass
R: friction (dissipation)
F: internal force (restoring force, etc)
F'(t): fluctuating force from heat bath

Power spectrum density (PSD) of the fluctuation force

$$S_{F'}(\omega) = 4k_{\mathrm{B}}TR$$

Displacement noise

- Transfer function approach
- Equivalently, the fluctuation of the system can be obtained from the response of the system

$$S_q(\omega) = \frac{4k_B T \, \mathrm{Re}[1/Z(\omega)]}{\omega^2} = -\frac{4k_B T \, \mathrm{Im}[H(\omega)]}{\omega}$$

- where Z(w) and H(w) are the impedance and force-to-displacement transfer function of the system

$$Z(\omega) = F(\omega)/\dot{q}(\omega), H(\omega) = q(\omega)/F(\omega)$$

Displacement noise

Question:

- Velocity damping of a pendulum

$$m\ddot{x} + \Gamma\dot{x} + m\omega_0^2 x = f$$

- Structural damping

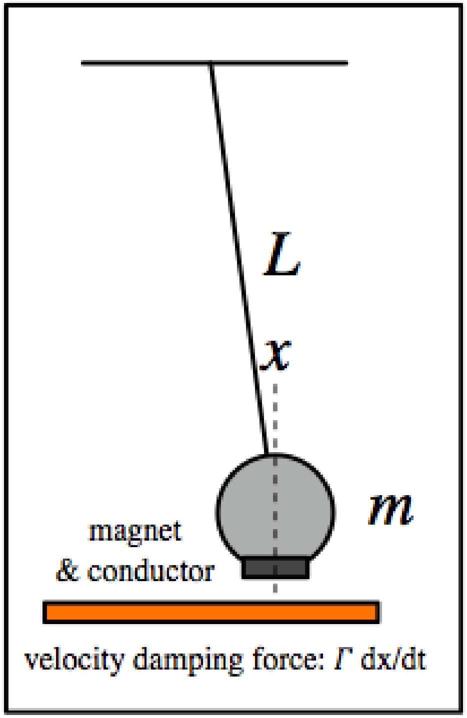
loss angle: 0 < $\phi \ll 1$

$$m\ddot{x} + m\omega_0^2(1 + \mathrm{i}\phi)x = f$$

- How does anti-spring change the thermal noise spectrum?

anti-spring parameter: 0<α<1

$$m\ddot{x} + m\omega_0^2(1 - \alpha + \mathrm{i}\phi)x = f$$



Displacement noise

- In some cases, calculating the system response is complicated (e.g. deformation of an elastic body)
- Systems response (impedance) at a certain freq:

$$Z(\omega) = F(\omega)/\dot{q}(\omega)$$

- Average rate of energy dissipation

$$W_{\rm diss} = \langle {\rm Re}(F){\rm Re}(\dot{q})\rangle$$

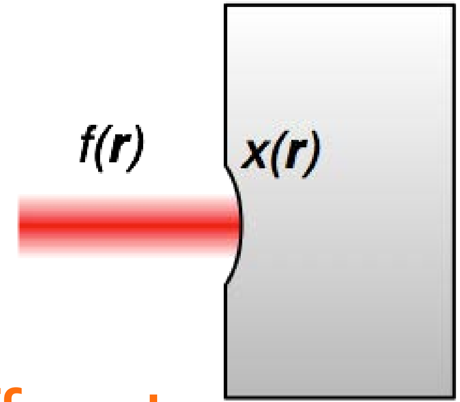
$$= \frac{1}{2}{\rm Re}[1/Z(\omega)]F_0^2 = \frac{1}{2}\frac{{\rm Re}[Z(\omega)]}{|Z(\omega)|^2}F_0^2$$

$$S_q(\omega) = \frac{4k_{\rm B}T}{\omega^2}{\rm Re}[1/Z(\omega)]$$

$$\boxed{S_q(\omega) = \frac{8k_{\rm B}TW_{\rm diss}}{F_0^2\omega^2}}$$

Displacement noise

- Sensing of the mirror surface deformation with a laser beam (with intensity profile of *f*(r))



- Apply periodic pressure with profile of *f*(r)

$$P(\mathbf{r}) = F_0 e^{i\omega t} f(\mathbf{r})$$

  This induces deformation of x(r) which is different from our sensing profile of *f*(r), but that's OK

- Calculate the rate of dissipation Wdiss

  analytically, using FEA, or etc

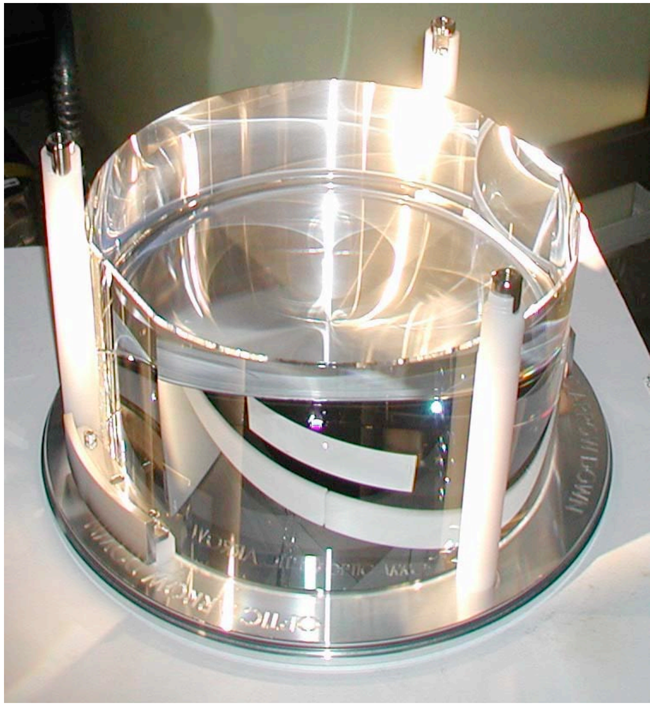
- Put this into the formula

$$S_x(\omega) = \frac{8 k_{\mathrm{B}} T W_{\mathrm{diss}}}{F_0^2 \omega^2}$$

Y. Levin PRD **57**, 659-663 (1998)

Displacement noise

- **Mirror substrate thermal noise**
  - Brownian motion
    Mechanical loss associated
    with the internal friction
    ⇔Thermally excited body modes
    Optical coating (high mechanical loss)
    will be limiting noise source in aLIGO



  - Thermo elastic noise
    Elastic strain & thermal expansion coefficient
    => cause heat distribution & flow in the substrate
    ⇔Temperature fluctuation causes mirror displacement
  - Thermo-refractive noise
    ⇔Temp. fluctuation causes fluctuation of refractive index

Displacement noise

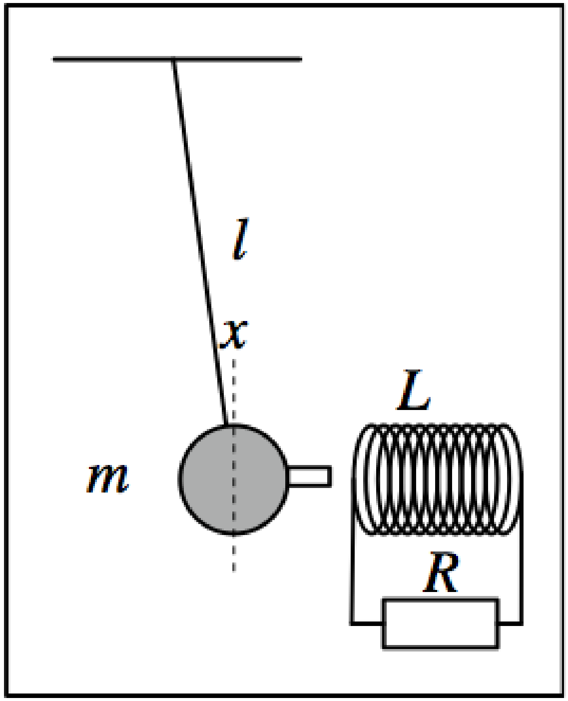
- Suspension thermal noise
  - Brownian motion
    Mechanical loss of the suspension fiber
    ⇔Thermally excited pendulum modes

  - Thermo elastic noise
    Elastic strain of the fiber & thermal expansion coefficient
    => cause heat distribution & flow in the fiber
    ⇔Temperature fluctuation causes mirror motion

<- Monolithic suspension
for high pendulum Q

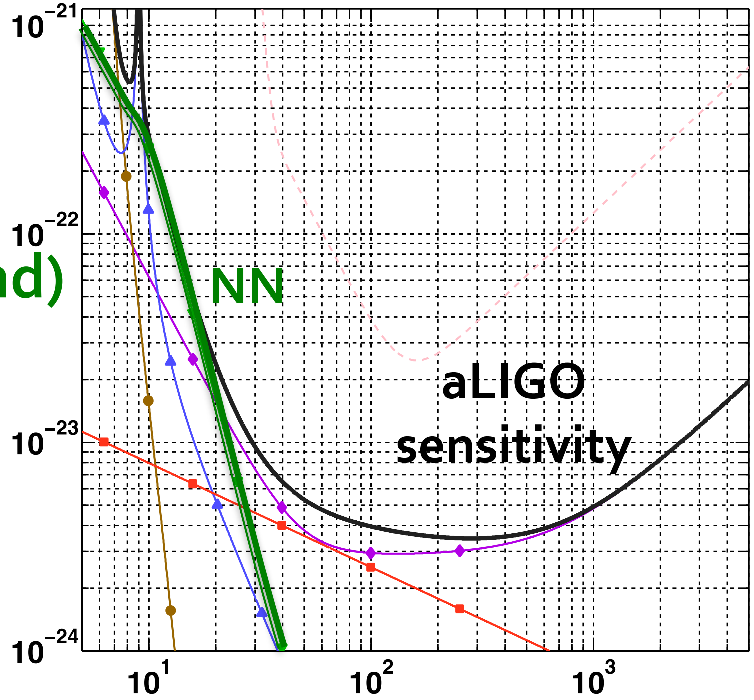
Displacement noise

- **Question**
  - **Induced current damping (electro-mechanical system)**
    1. How does the Q factor of the system depend on R?
    2. How much is the thermal noise displacement of the mass?
    3. How does the thermal noise of the resister shakes the mass?
    4. How are the above questions with a capacitive coupling instead of the coil?
  - **Cold damping**
    1. If the resister is cooled, how does the thermal noise motion change?
    2. Is the pendulum actually cooled? Down to what temperature?
    3. How fast the pendulum recovers the original temperature once R is returned to the room temp.?



Displacement noise

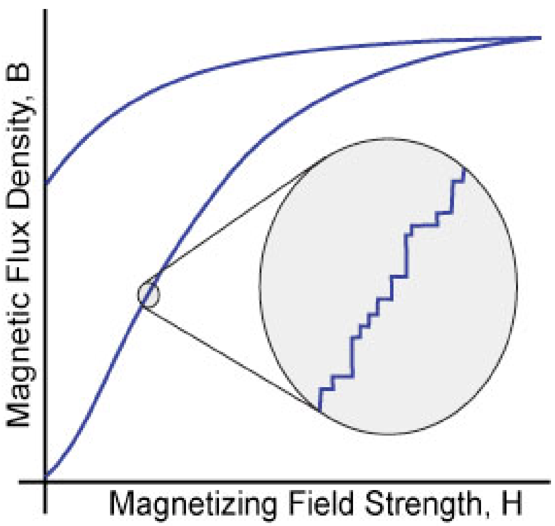
- Newtonian Gravity noise
  - Mass density fluctuations around the test masses
    => test mass motion via gravitational coupling
  - Dominant source of Newtonian noise
    = Seismic surface wave
  - Mitigation
    1) Going to quiet place (underground)
    2) Feedforward subtraction
    3) Passive reduction by shaping
       local topography

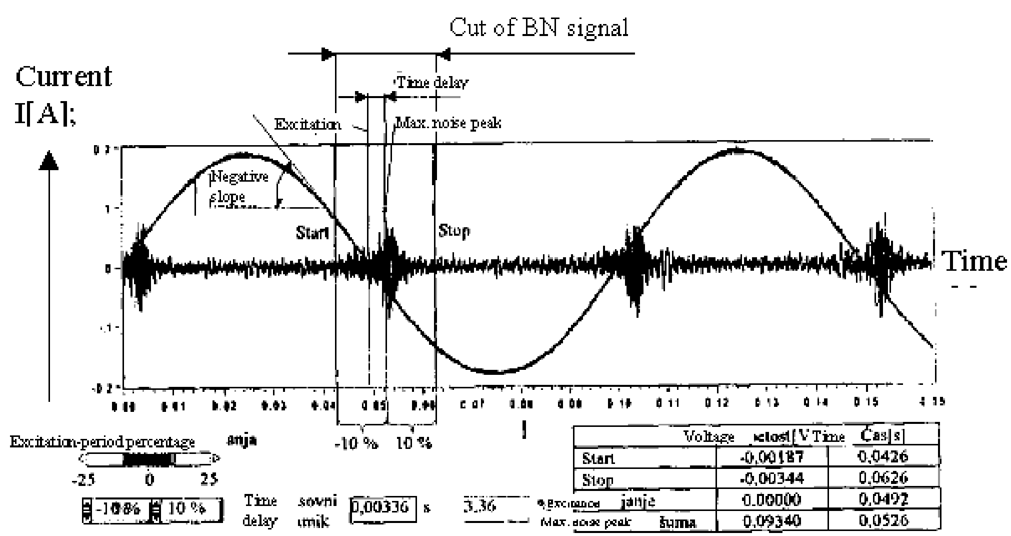


J Driggers, et al, PRD 86, 102001 (2012)
J Harms, et al, Class. Quantum Grav. 31 185011 (2014)

Displacement noise

- Mechanical upconversion noise
  - Large low frequency (f < 1Hz) motion
    => upconverted to 10~100Hz motion via nonlinear processes
  - Barkhausen noise
    => low freq mirror actuation cause BH noise and upconversion
    Select better magnet materials (e.g. SmCo)



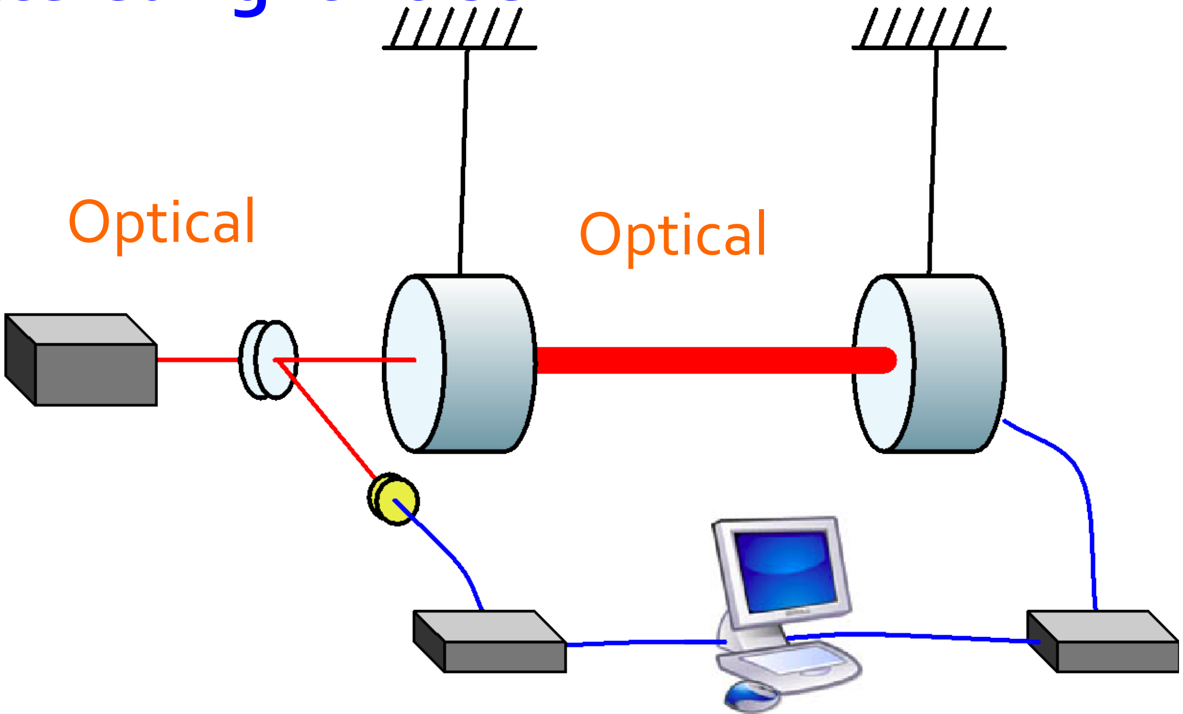


https://www.nde-ed.org/

Optical noises

Optical noises

- Noises that contaminate the readout signal
  - Quantum noises (shot noise, radiation pressure noise)
  - Laser technical noises (frequency/intensity noise)
  - Modulation noises
  - Scattered light noise



Optical noises

- Quantum noises: Shot noise
  - Noise due to photon counting statistics
  - N detected photon => standard deviation $\sqrt{N}$
  - Increasing the incident power P_{in},
    => The shot noise is increased by $\sqrt{P_{in}}$
    => The signal amplitude is increased by P_{in}
  - In total, the signal-to-noise ratio is improved by

$$\mathrm{SNR} \propto \sqrt{P_{\mathrm{in}}}$$

Optical noises

- **Quantum noises: Shot noise**
  - Photon shot noise associated with photodetection

$$i_{\rm shot} = \sqrt{2 e i_{\rm DC}} \ \ [{\rm A}/\sqrt{\rm Hz}]$$

  - Michelson interferometer

$$i_{\rm DC} = \frac{\eta P_{\rm in}}{h\nu} \frac{1 - \cos\delta\phi}{2} \ \ [{\rm A}]$$

$i_{\rm DC}$: DC Photocurrent
η : PD Quantum Efficiency
ν : Optical Frequency
at the limit of dϕ->0

$$i_{\rm shot} / \frac{d i_{\rm DC}}{d\phi} = \sqrt{\frac{2h\nu}{\eta P_{\rm in}}} \ \ [{\rm rad}/\sqrt{\rm Hz}]$$

Shot-noise limit of the Michelson phase sensitivity

  - Michelson response (@DC)

$$\frac{\delta\phi}{h_{\rm GW}} = \frac{4\pi L\nu}{c} \ \ [{\rm rad}/{\rm strain}]$$

Michelson Strain Sensitivity

1.3×10^{-20} 1/sqrtHz @1W

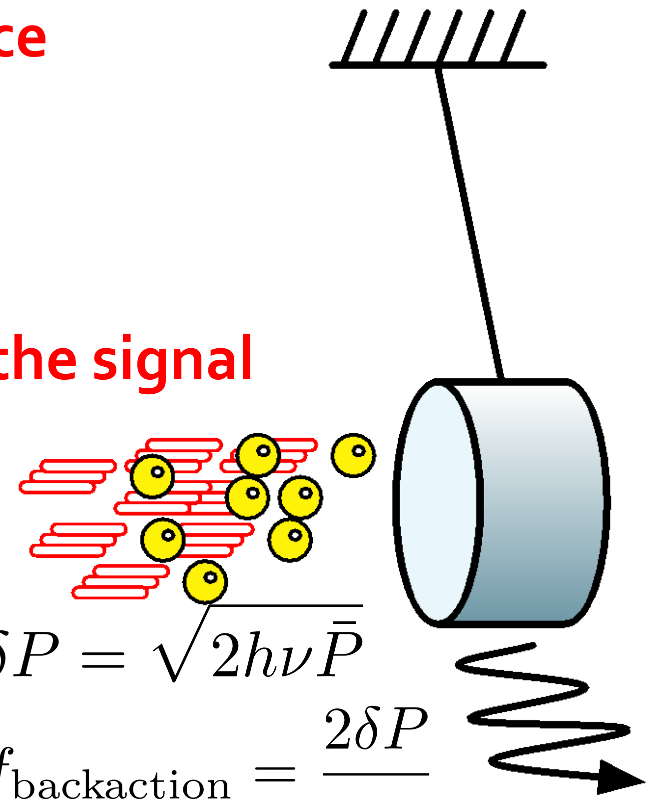
Optical noises

Supplemental slide ~ Shot noise derivation

- Take an average of Current $I(t)$ for a period of T, and sample it every T.
- Number of photons in this period T is $N = \bar{I}T/e$.
- Fluctuation of photon number in T is $\sigma_N = \sqrt{N}$. cf Poisson statistics
- Thus, the standard deviation (RMS) of $\bar{I}$ is $\sigma_I = e\sqrt{N}/T = \sqrt{e\bar{I}/T}$
- Think about the transfer function of this box car average filter. It is $H(f) = \mathrm{sinc}(\pi f T)$
- Parsevals theorem: $\sigma_I = \int_0^\infty H(f)^2 i_s^2 df$, where i_s is the linear power spectrum density of the current (white spectrum).
- According to the above integration, $i_s = \sigma_I\sqrt{2T}$.
- Therefore we obtain $i_s = \sqrt{2e\bar{I}}$.

Optical noises

- Quantum noises ~ Radiation pressure noise
  - Photon number fluctuation in the arm cavity
    => Fluctuation of the back action force
  - Quantum noise of the input laser
    => Common noise for two arms
    => cancelled and does not appear in the signal
  - Vacuum fluctuation injected
    from the dark port
    => Differentially power fluctuation
    => Cause the noise in the GW signal



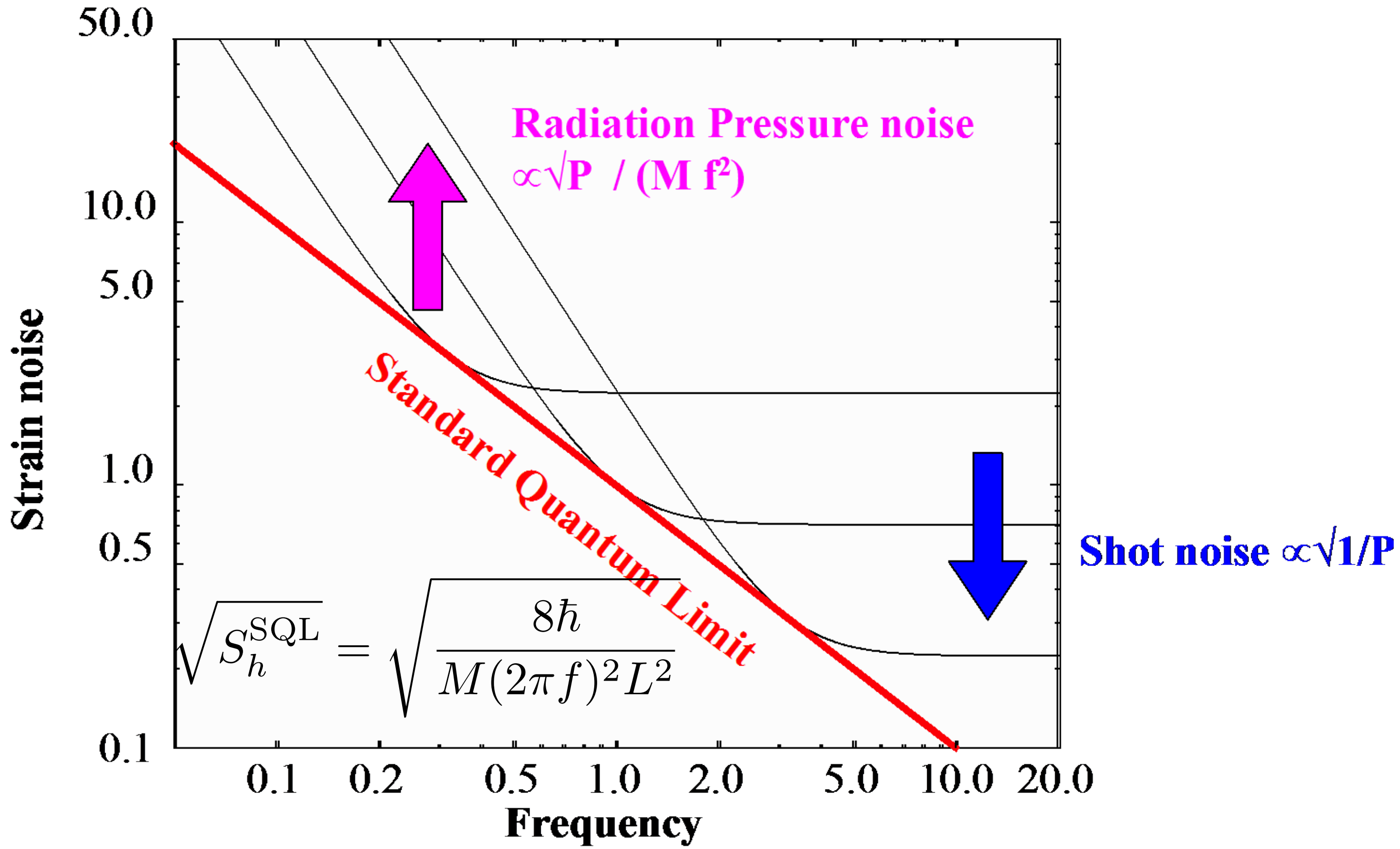
$$\delta P = \sqrt{2h\nu\bar{P}}$$

$$f_{\mathrm{backaction}} = \frac{2\delta P}{c}$$

$$\tilde{x} = \frac{f_{\mathrm{backaction}}}{M\omega^2}$$

Optical noises

- Quantum noises
  - Standard Quantum Limit (SQL)



- Trade-off Between Shot Noise and Radiation-Pressure Noise
- Uncertainty of the test mass position due to observation

Optical noises

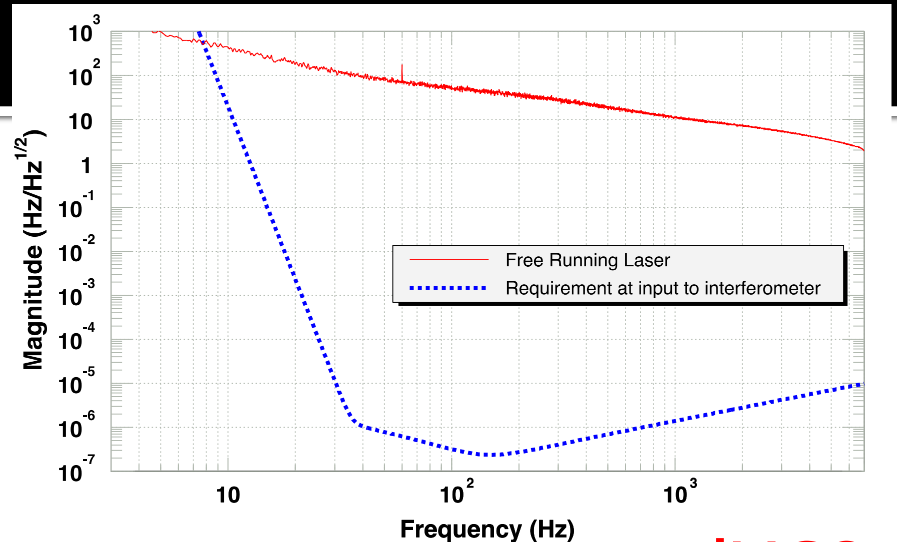
- Laser frequency noise
  - Laser wavelength ($\lambda = c / \nu$)
    = reference for the displacement measurement
  - Optical phase $\phi = 2 \pi \nu L / c$
    $d\phi = 2 \pi / c (L \, d\nu + \nu \, dL)$ <= indistinguishable

$$\frac{dL}{L} = \frac{d\nu}{\nu}$$

  - dL/L target 10^{-24}
    => $d\nu = 10^{-24} \times 3$ THz (1064nmYAG laser)
    $= 3 \times 10^{-10}$ Hz/rtHz

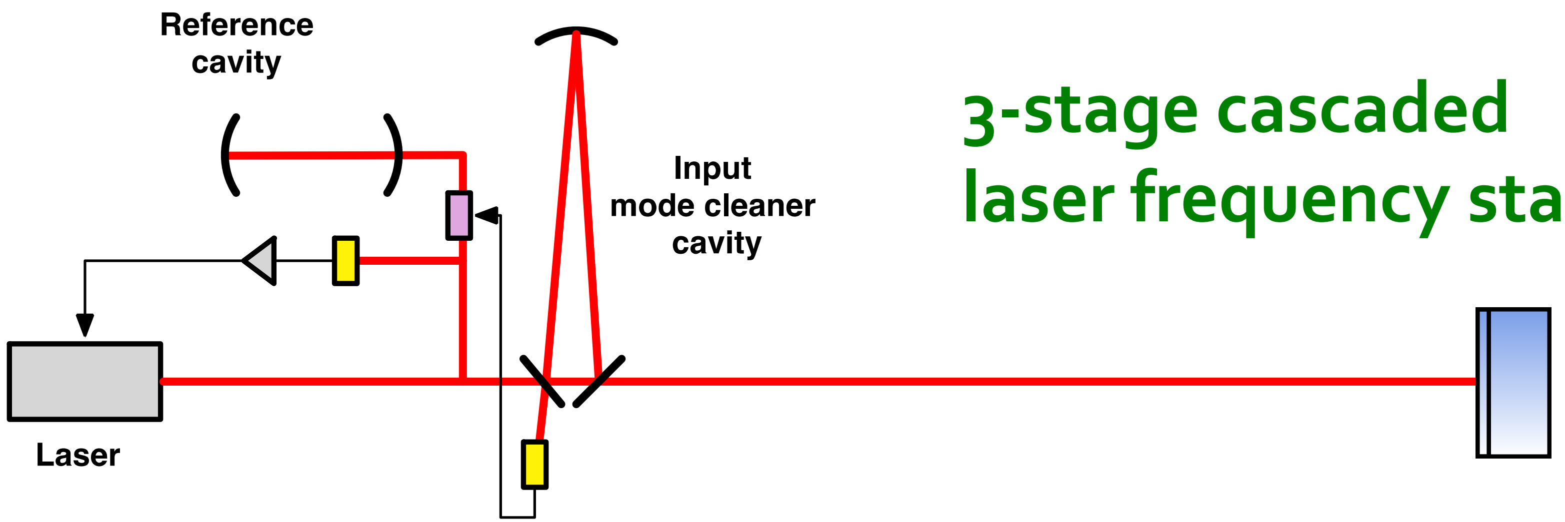
Optical noises

- Laser frequency noise
  - Target: $d\nu_{eff} = 3 \times 10^{-10}$ Hz/rtHz
  - Laser stability
    $d\nu$ = 10~100 Hz/rtHz @100Hz



iLIGO

3-stage cascaded
laser frequency stabilizatio



Michelson's differential sensitivity provides
Frequency noise cancellation of 1/100~1/1000
"Common Mode Rejection"

Optical noises

- Laser intensity noise
  - Relative Intensity Noise (RIN): dP/P
  - Sensor output V = P x

    => dV = P dx + x dP <= indistinguishable

$$\frac{dx}{x_{\mathrm{offset}}} = \frac{dP}{P}$$

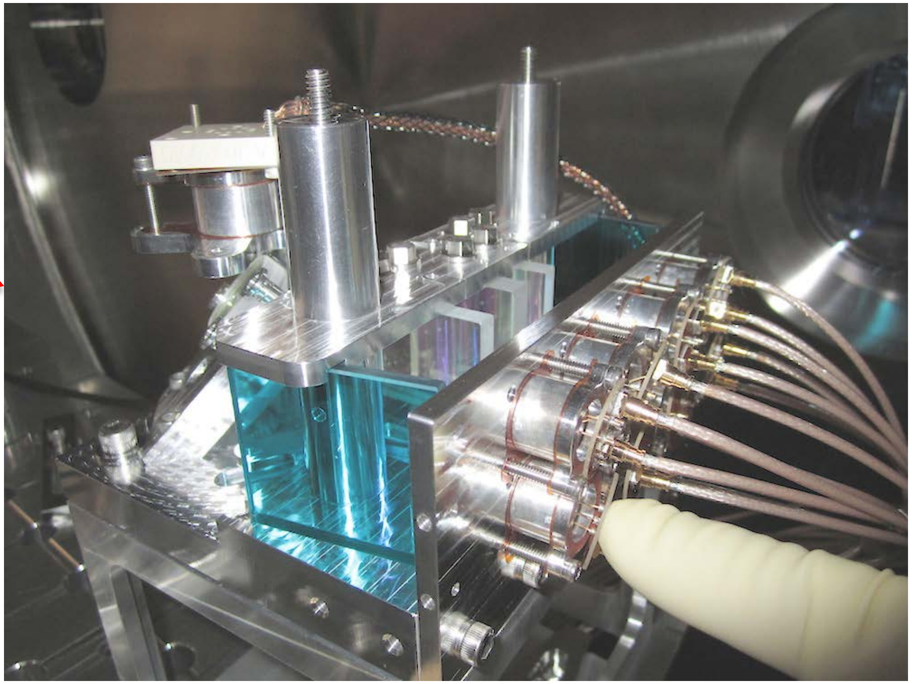
  - Requirement: RIN = 10^{-9} $1/\sqrt{\mathrm{Hz}}$

Optical noises

- **Laser intensity noise ~ intensity stabilization**
  - Requirement: RIN = 10^{-9} $1/\sqrt{\text{Hz}}$
  - 2-stage cascaded intensity stabilization control
  - Challenge: requires 300mA of photodetection

Shot noise limited RIN $$\frac{i_{\rm shot}}{i_{\rm DC}} = \frac{\sqrt{2ei_{\rm DC}}}{i_{\rm DC}} = \sqrt{2e/i_{\rm DC}}$$

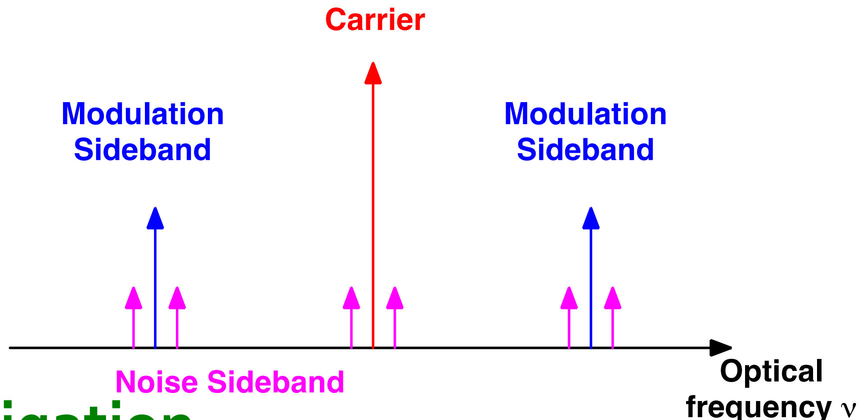
  - In-vacuum 8-branch Photodiode array

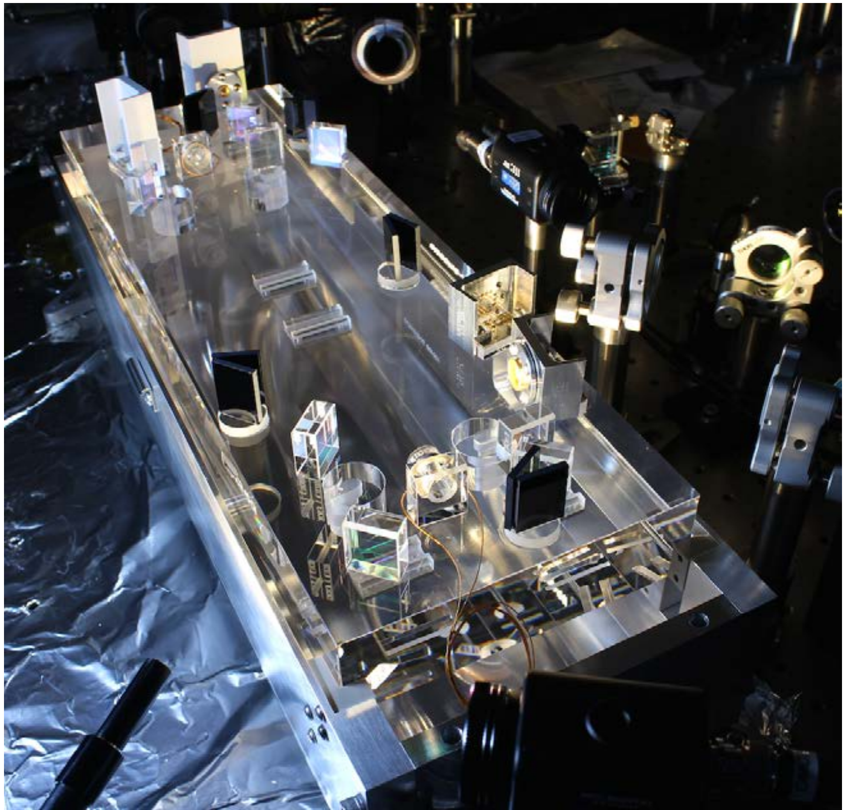


P. Kwee et al,
Optics Express **20** 10617-10634 (2012)

Optical noises

- Modulation noises
  - RF Residual Amplitude Modulation
  - Modulation Oscillator Phase Noise
  - Modulation Oscillator Amplitude Noise
- Produce noise sidebands on the modulation sidebands



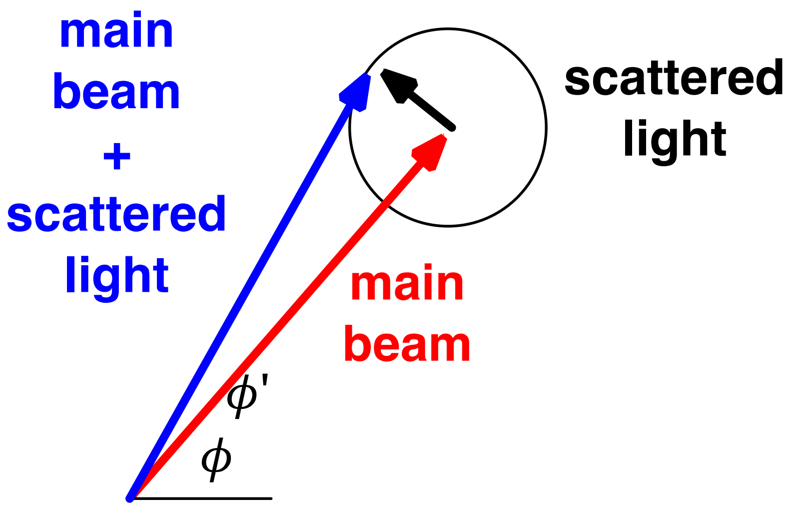


- Mitigation
  - For the GW signal:
    Use DC readout and eliminate them by an "output mode cleaner cavity"

J. Camp et al JOSA A **17** 120-128 (2000)

Optical noises

- Scattered light noise
  - Scattered light recouples to the interferometer beam with an arbitrary phase
    => causes amplitude and phase fluctuation
  - Two effects:
    1. Small motion regime: linear coupling of the phase fluctuation
    2. Large motion regime: low freq large motion of the scattering object => upconversion via fringe wrapping
  - Mitigation
    - Reduce scattered light
    - Vibration isolation of the scattering object



Electrical noises

Electrical noises

- **General rules for electrical noises**
- **Electrical noise in photo detection**
- **Digitization noise (ADC/DAC) / Aliasing**
- **Control noise**
- **Actuator noise**

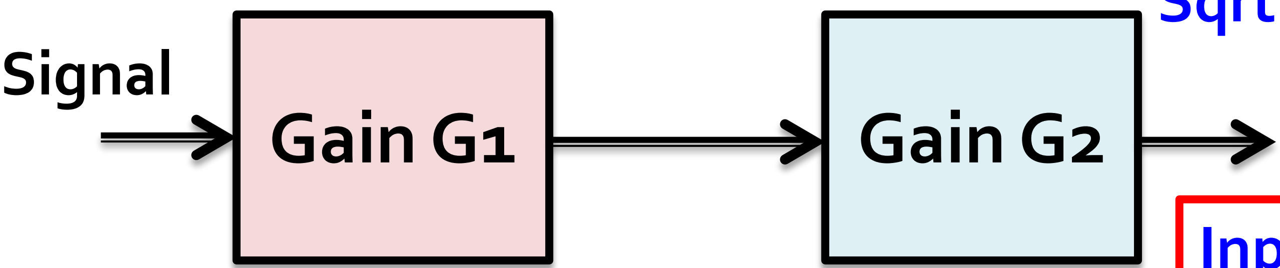
General rules for electrical noises

- Low noise amplification at the beginning
- Give necessary gain as early as possible
- Don't attenuate (and amplify again)

input noise: v_1

input noise: v_2

output noise Vout:
$Sqrt[(v_1 G_1 G_2)^2+(V_2 G_2)^2]$



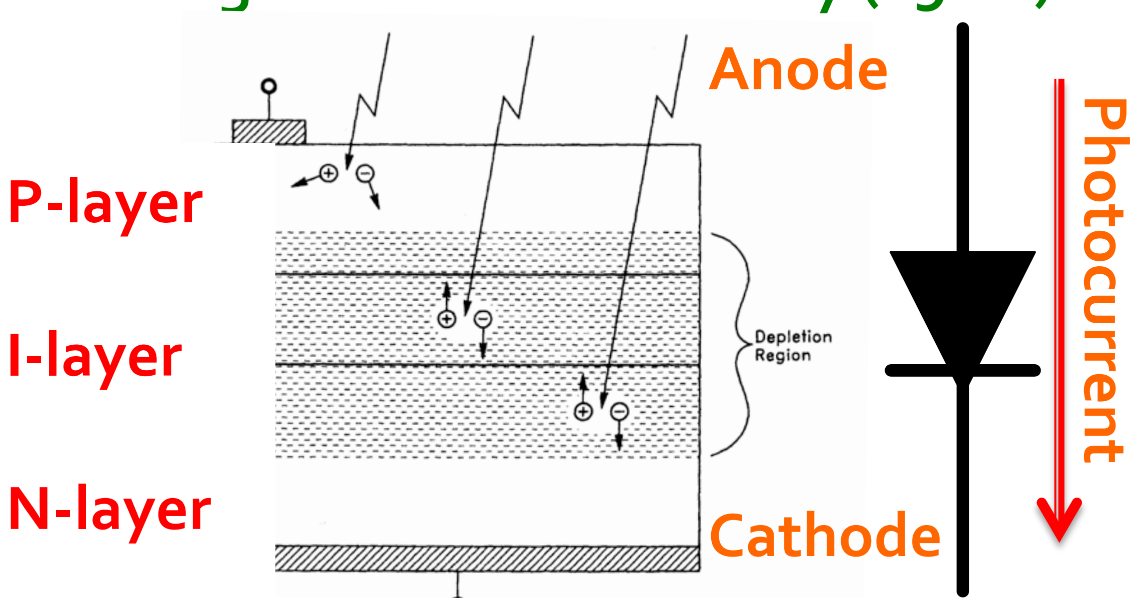
output noise: $v_1 G_1$

Input equivalent noise
Vout / (G_1 G_2)
$= Sqrt[v_1^2+(v_2/G_1)^2]$

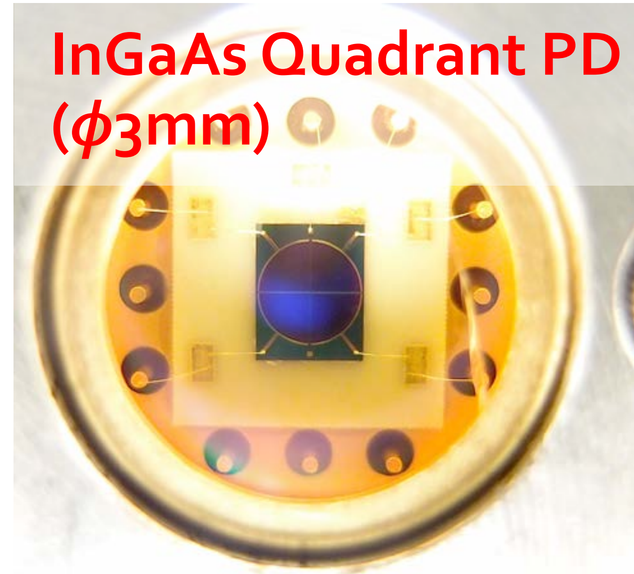
- Lessons
  - The input referred noise is determined by v_1
  - It won't become better by the later stages
  - If G_1 is big enough, we can ignore the noise of later stages

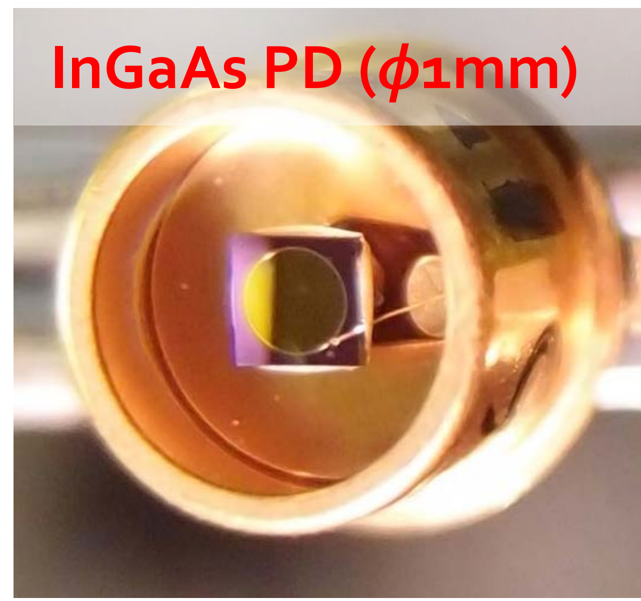
Noise in photodetectors

- Photodiodes
  - PIN photodiodes
    (InGaAs for near IR, Si for visible)
    - Good linearity
    - Low noise
    - High Quantum Efficiency (>90%)



“Photodiode Amplifiers”, J. Graeme (McGrawHill 1995)





Noise in photodetectors

- Photodetectors are the first electrical block of the control chains
  - It is important to have
    low input-referred current noise

- Photo detection
  - AF (Audio Frequency 0~100kHz)
    - Plenty of light (photocurrent ~mA)
      Not a big electrical issue
  - RF (Radio Frequency 10~200MHz)
    - Large diode aperture -> high RF noise
      Need careful consideration

Noise in photodetectors

- Noise in photodiodes
  - Photodiode equivalent circuit
    - Shunt Resistance R_D (~100MΩ) Usually not a problem
    - Junction Capacitance C_D (1pF~1nF)
    - Series Resistance R_S (1Ω~100Ω)

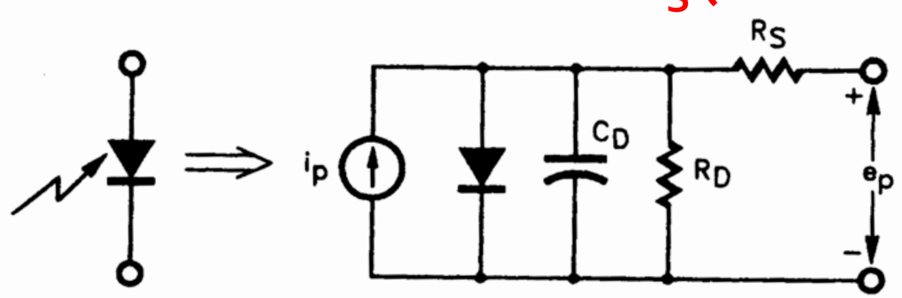


Figure 1.3 The circuit model of a photodiode consists of a signal current, an ideal diode, a junction capacitance, and parasitic series and shunt resistances.

input referred noise current

$$i_{Rs} \sim \omega C_d \sqrt{4 k_B T R_s}$$

The diode aperture size needs to be ~mm => Cd tends to be big.

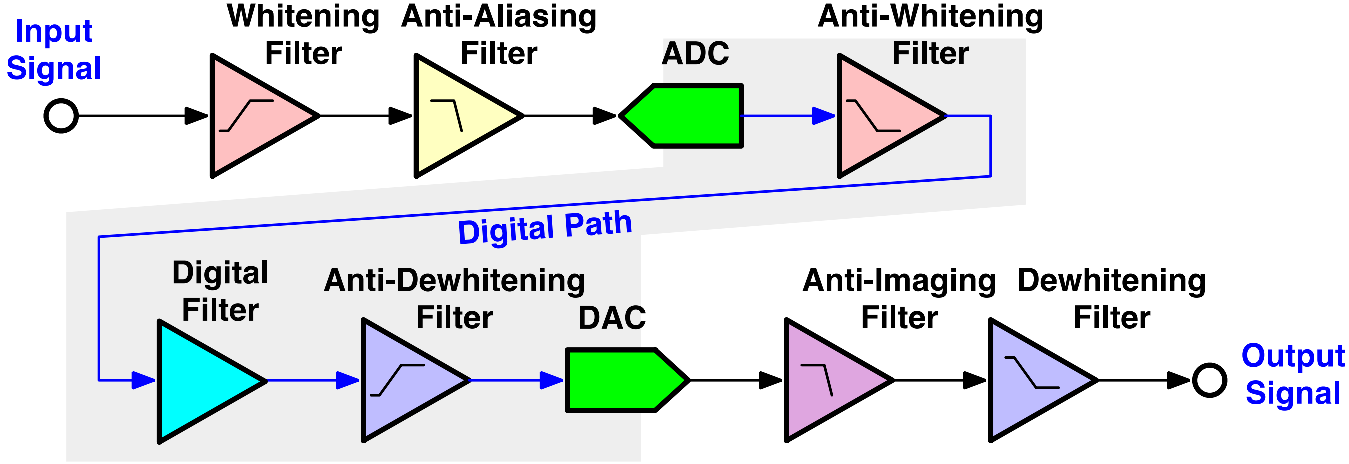
2mm InGaAs PD: Rs~10Ω, Cd~100pF

=> i_Rs = 20 pA/sqrtHz @100MHz

(equivalent to the shot noise of 1mA light ~ 1.3mW@1064nm)

Analog/Digital interface

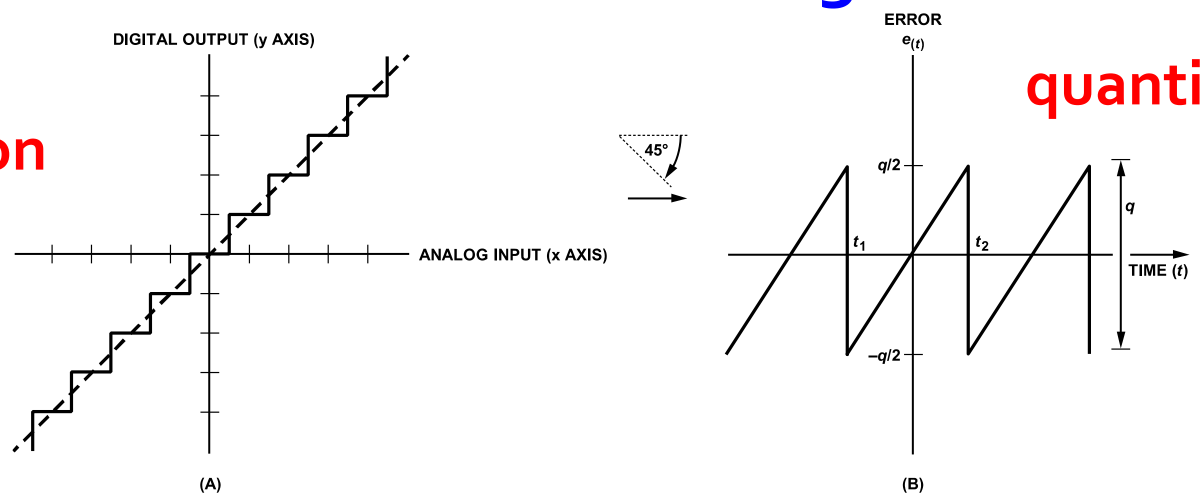
- Restriction of signal digitization
  - Voltage quantization: quantization noise
    => limited dynamic range
    => Requires whitening/dewhitening filters
  - Temporally discrete sampling: aliasing problem
    => limited signal bandwidth
    => Requires anti-aliasing (AA) / anti-imaging (AI) filters
  - Typical signal chain



Digitization (Quantization) noise

- Analog signals (~+/-10V) -> Digital signal
  - Digitized to a discrete N bit integer number

quantization



quantization error

ansfer Function (A) and Ideal N-Bit ADC Quantized Noise (B)

http://www.analog.com/static/imported-files/tutorials/MT-229.pdf

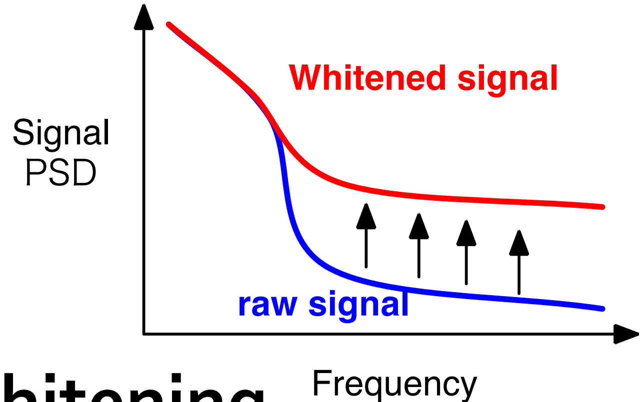
- Quantization causes a white noise $V_n = \frac{\Delta}{\sqrt{12}} \; [\mathrm{V}/\sqrt{\mathrm{Hz}}]$

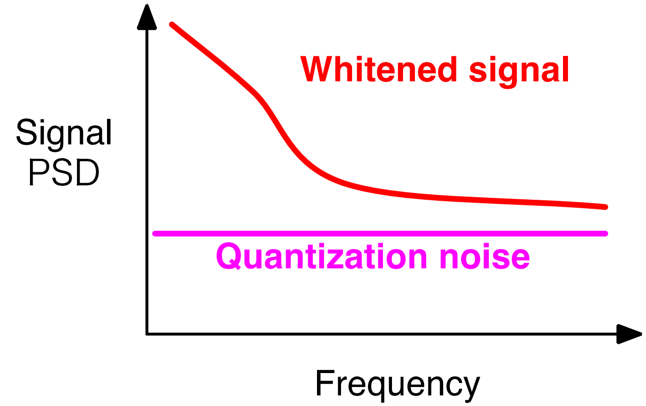
  e.g. +/-10V 16bit => Δ = 0.3mV => Vn ~ 100 μV/sqrtHz

  cf. Input noise of a typical analog circuit 10nV/sqrtHz

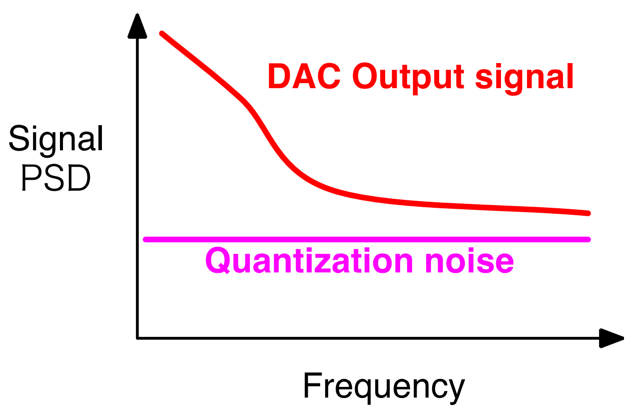
Digitization (Quantization) noise

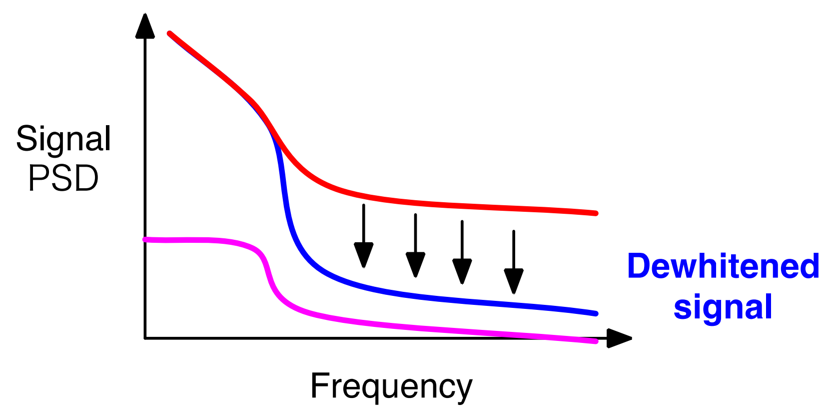
- **Whitening**
  - Amplify a signal in the freq band where the signal is weak





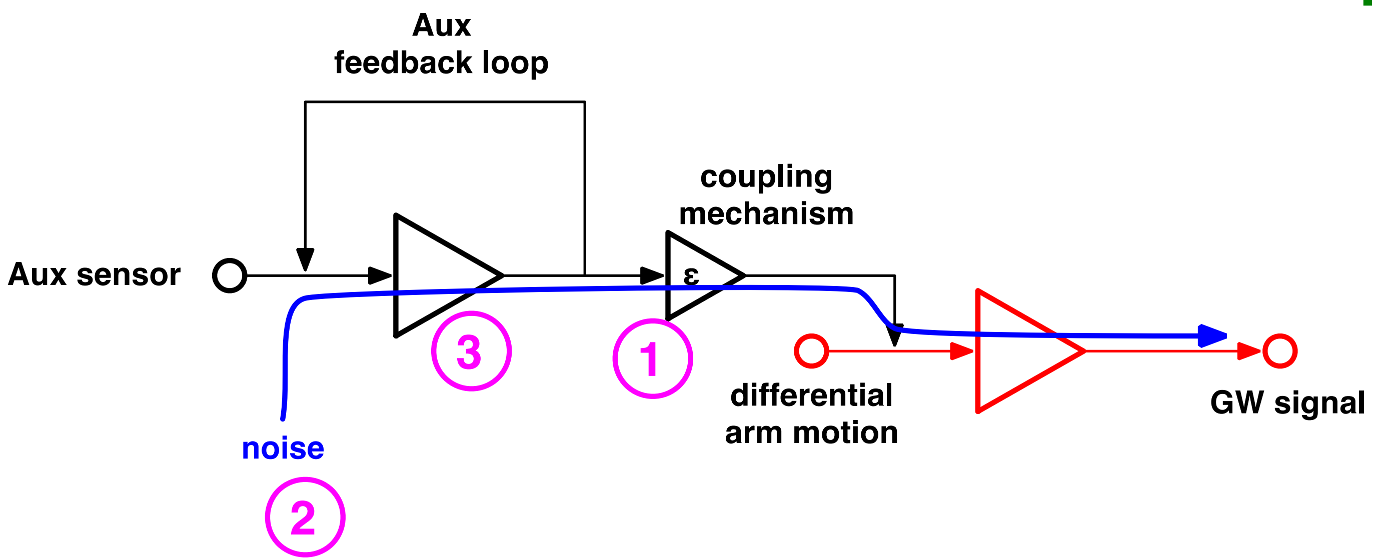
- **Dewhitening**
  - Amplify a signal in the freq band where the signal is weak





Control induced noise

- Noise couplings from auxiliary loops
  - e.g. Angle control feedback
    -> noise injection to the GW channel
  - Mitigation
    1) Make the coupling smaller
    2) Make the noise itself smaller
    3) Limit the control bandwidth of the aux loop



Actuator noise

- Actuator noise appears in the GW signal as an external disturbance

  - Mitigation
    1) Make the noise itself smaller
    2) Make the actuator response smaller

  - We need to keep sufficient actuator strength for lock acquisition
    => Transition to a low-noise mode after achieving lock

Summary

Summary

- Summary
  - There are such large number of noises
  - They are quite omnidisciplinary
  - Even only one noise can ruin our GW detection

  - GW detection will be achieved by
    - Careful design / knowledge / experience
    - Logical, but inspirational trouble shooting
  - Noise "hunting"