EE380 (Control Systems)

Lecture 3: Experiment 2

Speed of PMDC motor tracks a reference sinusoid

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Contents

Part I

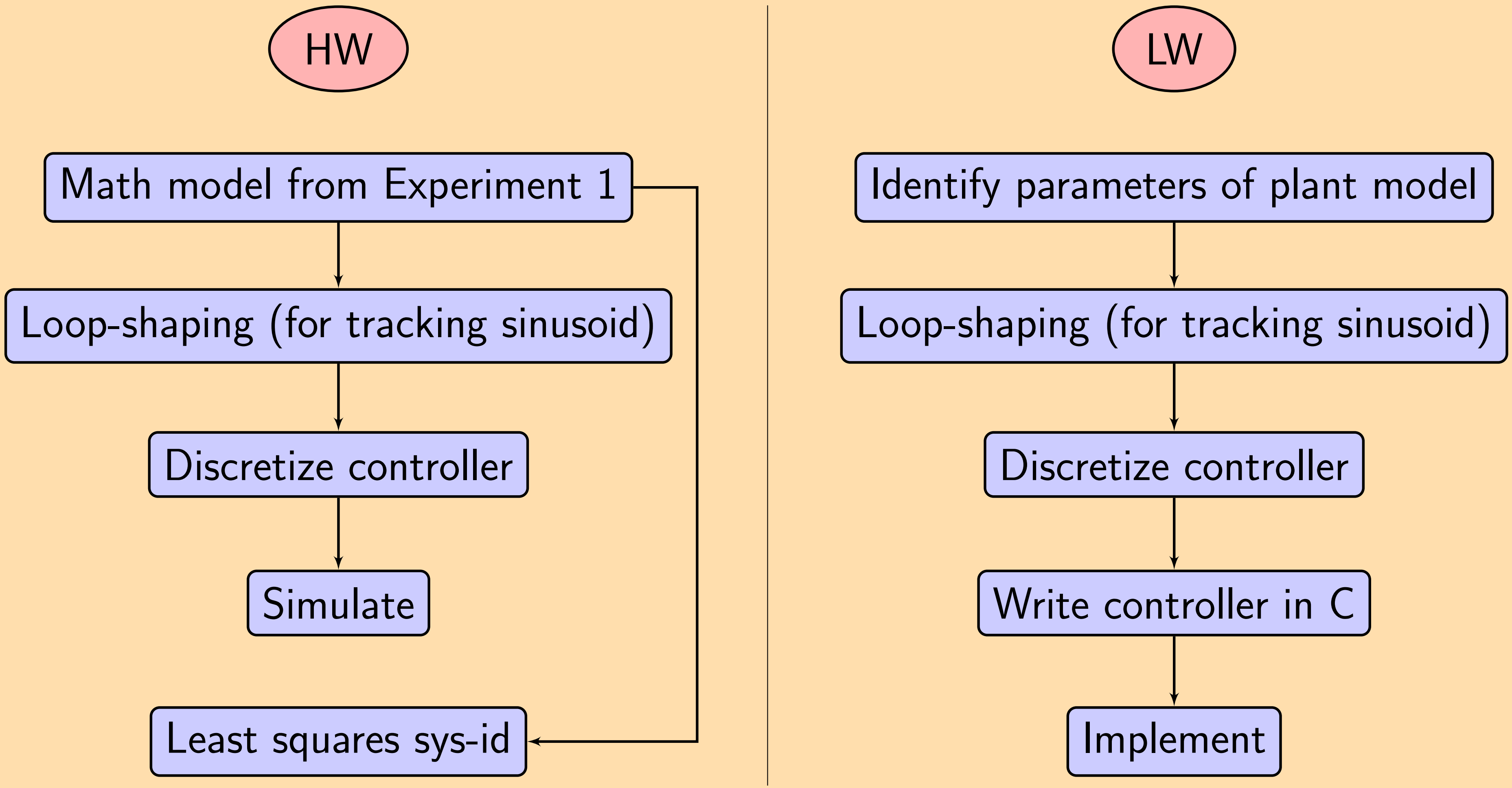
Procedure

1 Outline of the experiment

Want speed of motor to track sinusoids. Steps:

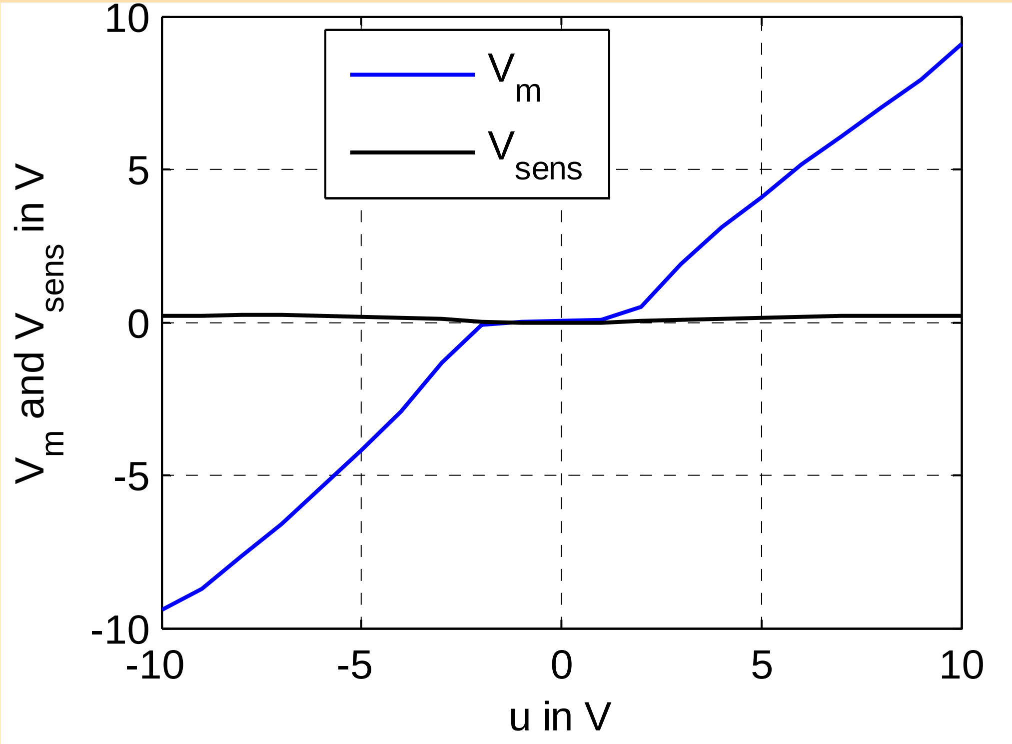
- Least squares system identification (sys-id).
- Recognition ~~and compensation~~ of plant's dead zone.
- Design controller using loop-shaping.
- Simulation on PC.
- Deployment on experimental setup.

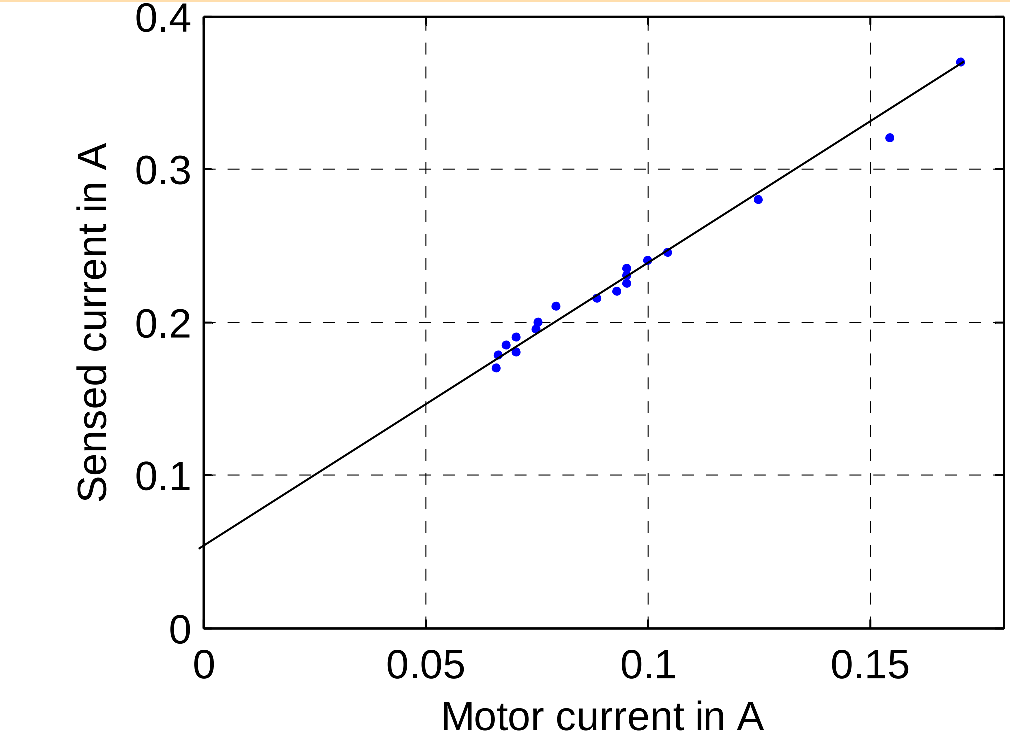
2 Homework (HW) vs. Lab work (LW)



- How well does sinusoid track reference sinusoid in theory & in practice?
- What control effort is needed for this tracking in theory & in practice?

3 Dead zone 1/2

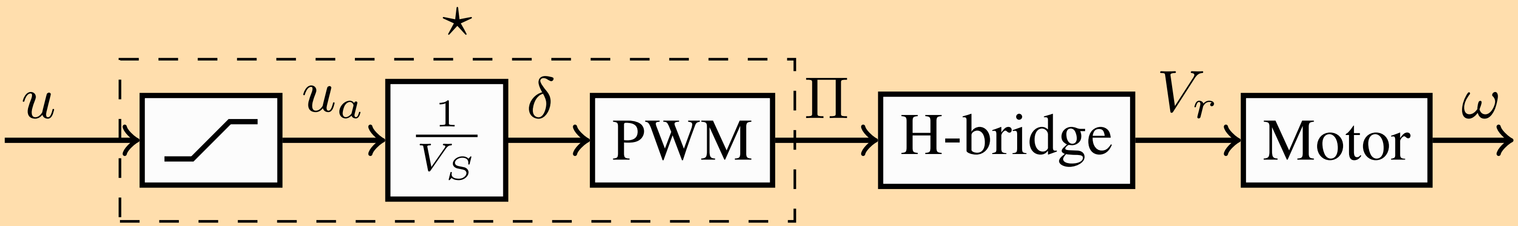




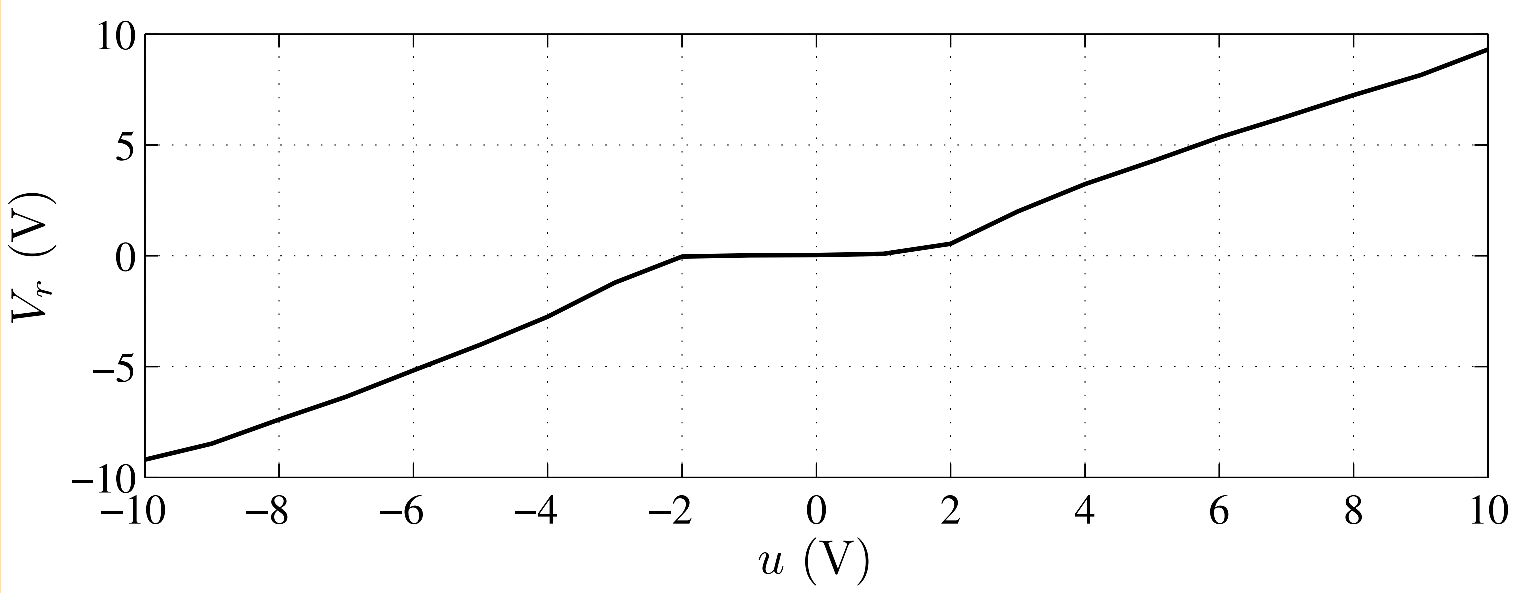
- Plots taken at $V_s = 12$ V.

- $V_m = 0$ when $|u| \leq 2$ V.
- $V_m \approx u$ when $u > 2$ V.

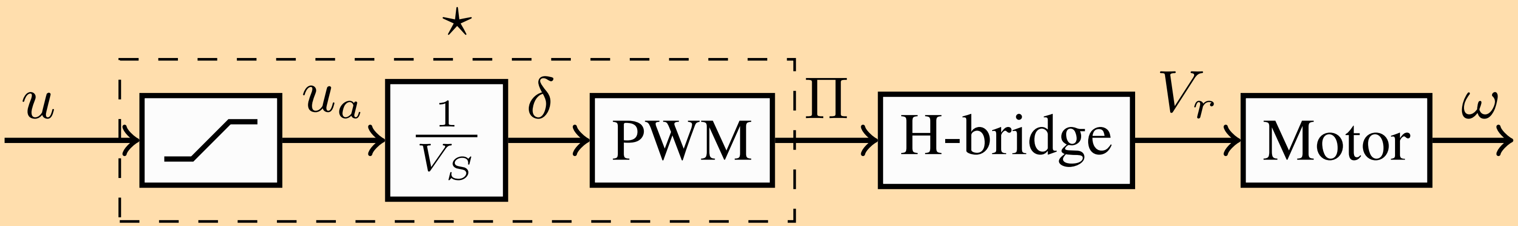
- V_m is same as V_r.
- V_{sens}: voltage across R_{sens}.



Another representation of deadzone:

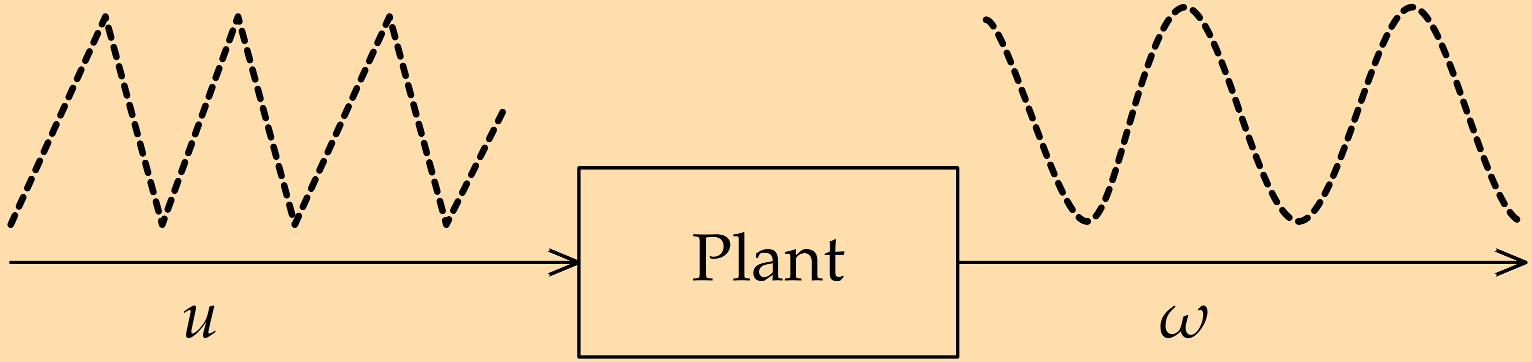


- V_r is same as V_m.



- Overcoming deadzone — topic of another experiment.

5 Least squares sys-id



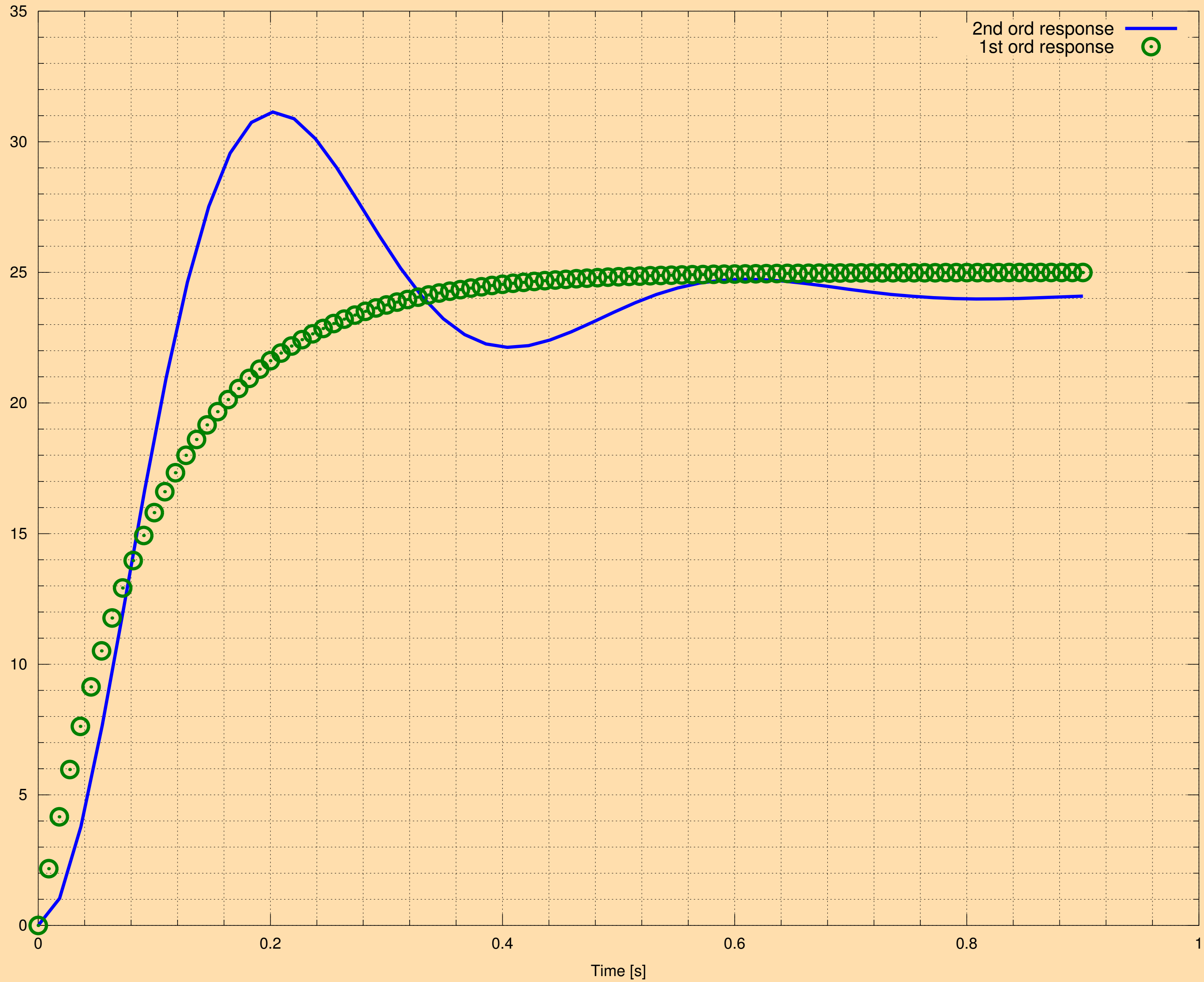
- Prepare setup to apply u in OL and collect ω from motor.
- Input $u(1), u(2), u(3), \ldots$ forming the rectangular/triangular/sinusoidal waveform.
- Collect $u(1), u(2), u(3), \ldots$ and $\omega(1), \omega(2), \omega(3), \ldots$ into `terminal.log`.

$\boldsymbol{u}$	$u(1)$	$u(2)$	$u(3)$	$\cdots$
$\boldsymbol{\omega}$	$\omega(1)$	$\omega(2)$	$\omega(3)$	$\cdots$

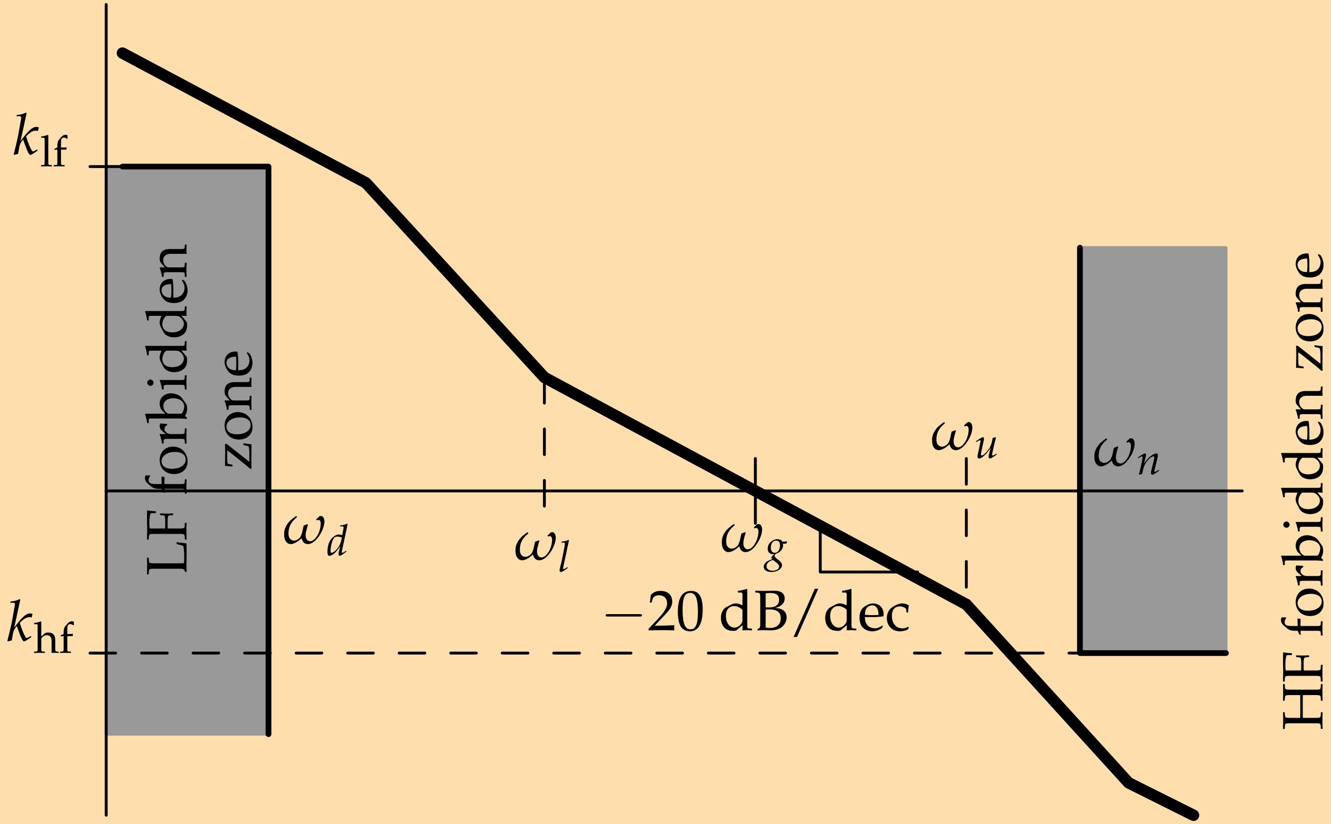
- Provided is `readSID.m` — Formed by replacing plots-related section of `readplot.m` with sys-id code from `sysid.m`.
- Execute `readSID.m` to obtain K, a, b of $\frac{K}{s^2+as+b}$.

6 Approximate 2nd order TF by 1st order TF

Generate a 1st order approxiation $G(s)$ to $\frac{K}{s^2+as+b}$. Can use an m-file.

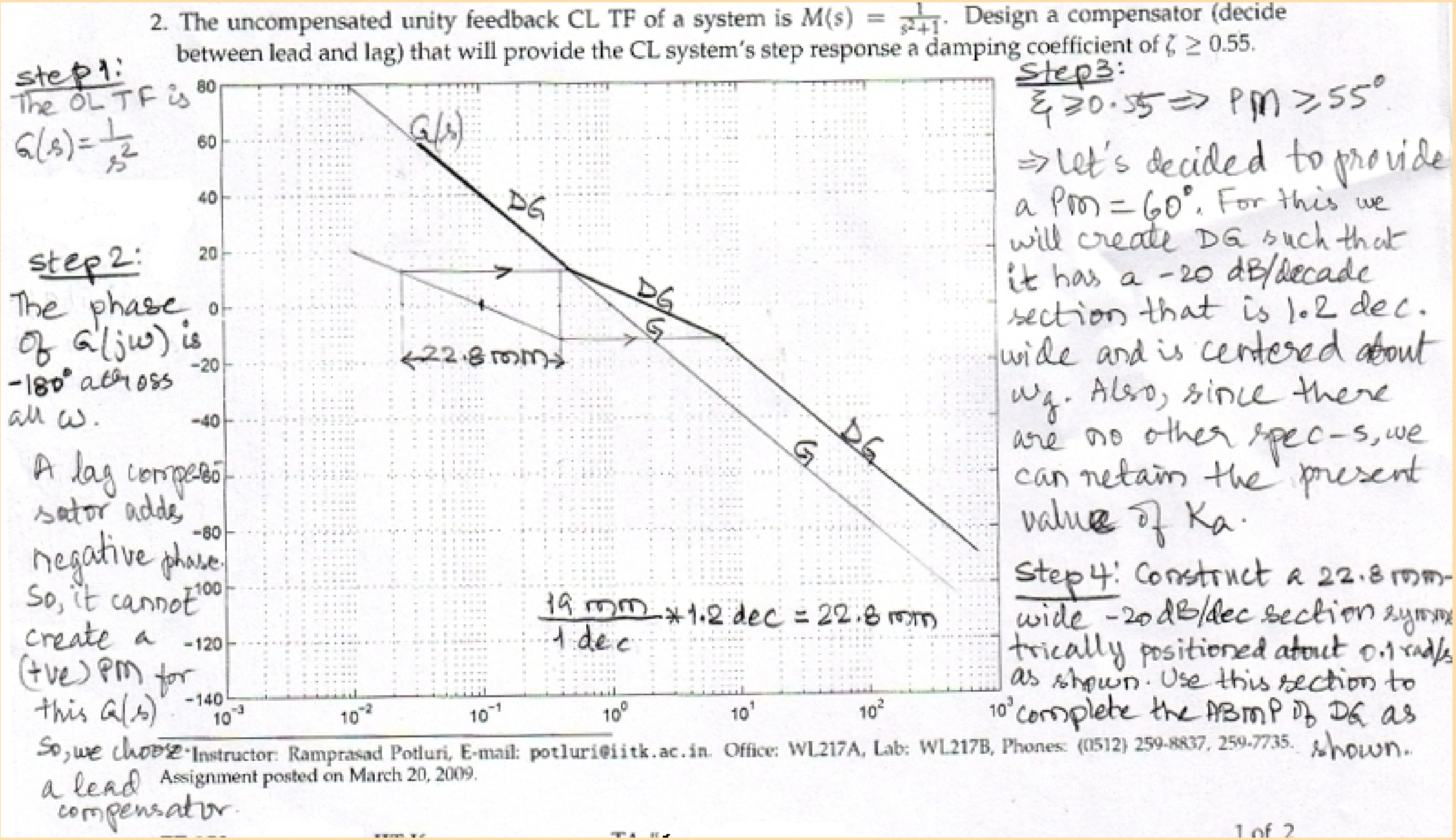


7 Loop-shaping (1/4): Typical G_{des}



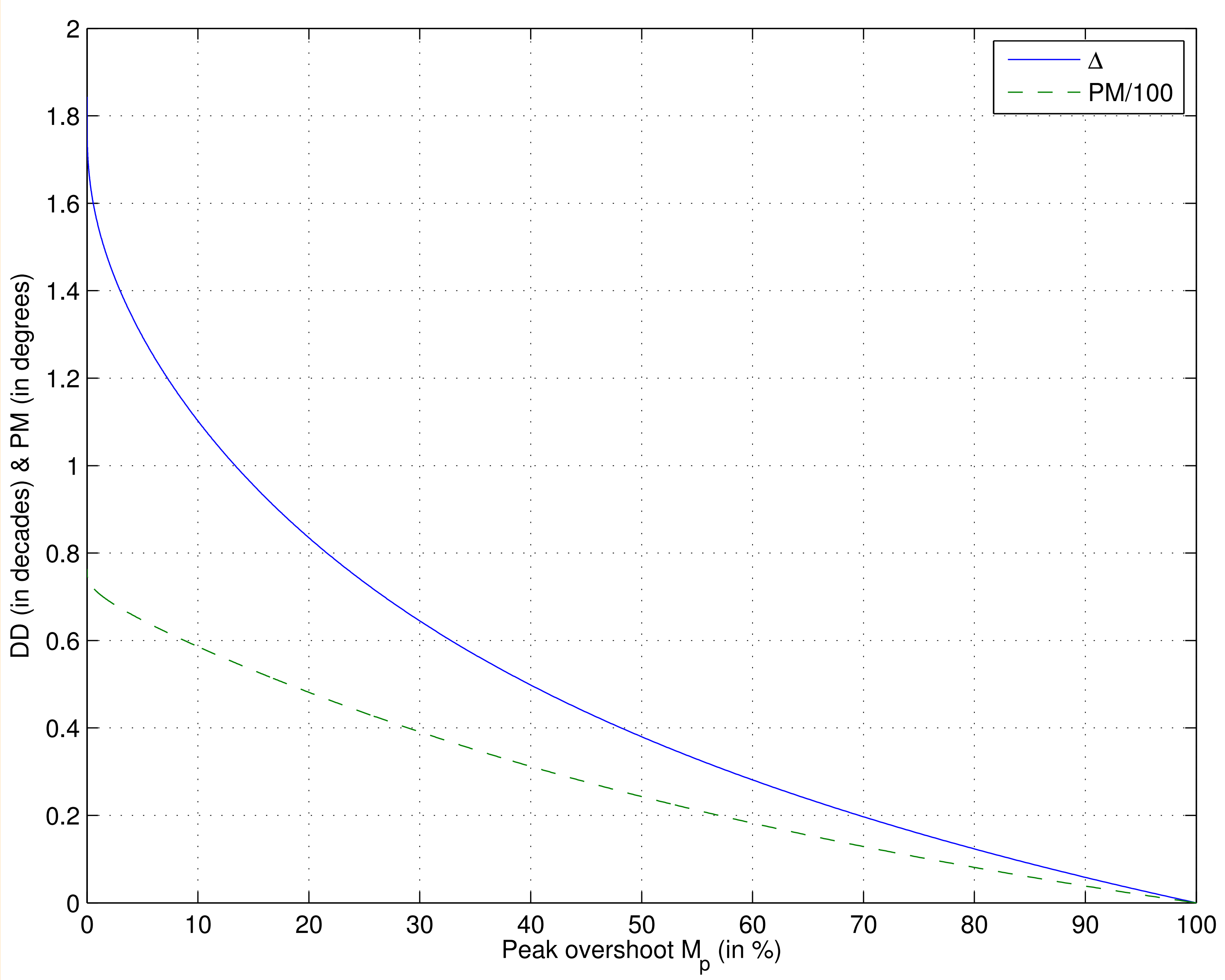
- In this experiment, ω_g may not matter; only ω_d does.
- Only a 1st order controller $D(s)$ is needed. Can use a 1st order desired loop shape for $D(s)G(s)$.

8 Loop-shaping (2/4): Example

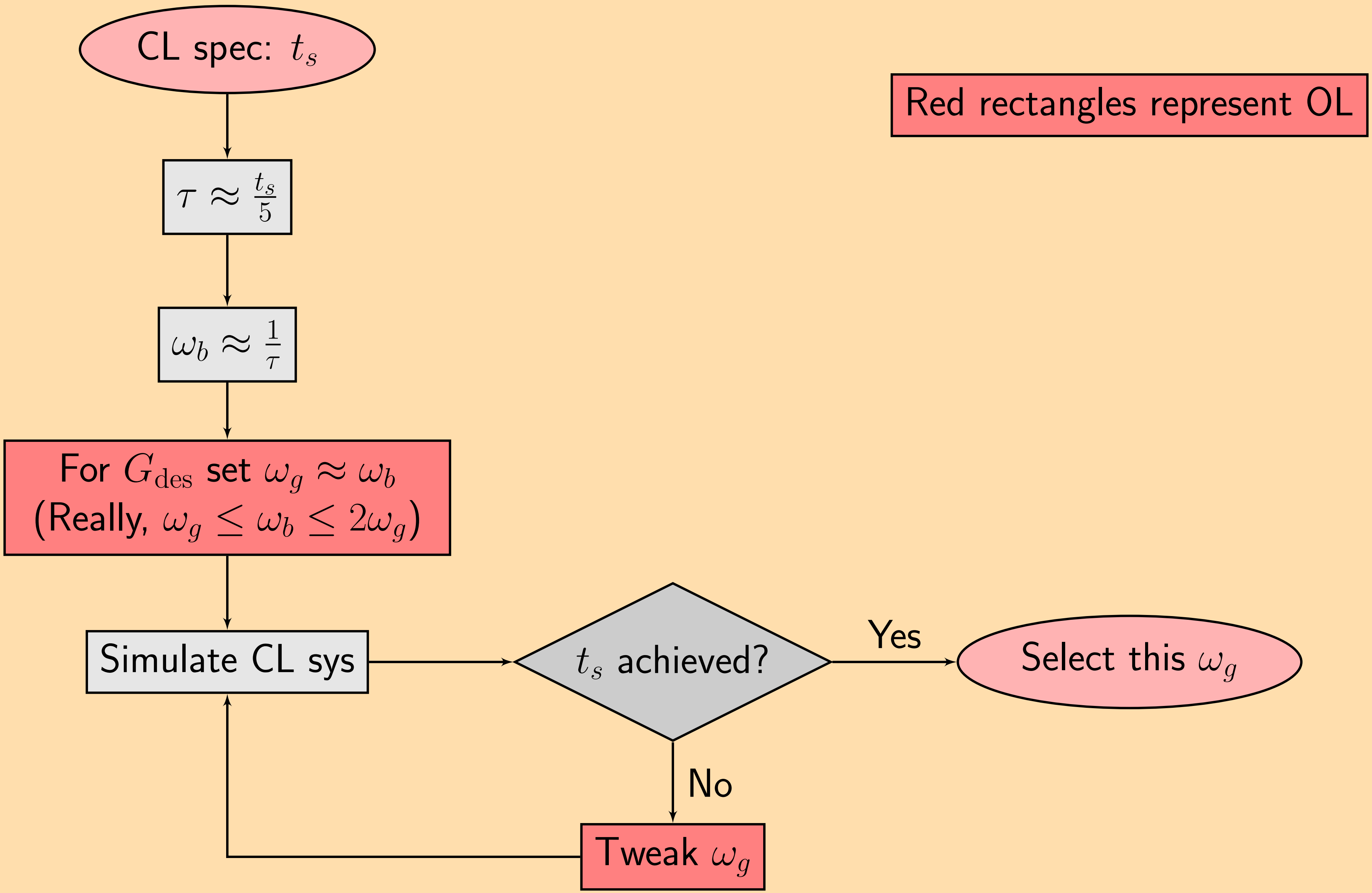


$$DG = G_{\text{des}}$$

9 Loop-shaping (3/4): $M_p \longleftrightarrow \mathrm{DD}$ (from EE250)



10 Loop-shaping (4/4): Determination of ω_g



11 Discretization

$$\frac{a_1 s + a_0}{s^2 + b_1 s + b_0}$$

Simulation diagram
(or `tf2ss`)

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -b_0 x_1 - b_1 x_2 + u \\
y &= a_0 x_1 + a_1 x_2
\end{aligned}$$

Euler's approximation

$$\begin{aligned}
x_{1(k+1)} &= x_{1(k)} + \Delta t \; x_{2(k)} \\
x_{2(k+1)} &= -b_0 \Delta t \; x_{1(k)} + (1 - b_1 \Delta t) x_{2(k)} + \Delta t \; u_k \\
y_k &= a_0 x_{1(k)} + a_1 x_{2(k)}
\end{aligned}$$

12 Simulate; LW: C code, Implement, Analyze

- Simulation: `simsine.m`
- Discretized controller $\longrightarrow$ C code:
- Implement: As in demo slides
- Analyze: Compare results

$$
\begin{aligned}
x_1(k+1) &= a_{11}x_1(k) + a_{12}x_2(k) + b_1u(k) \\
x_2(k+1) &= a_{21}x_1(k) + a_{22}x_2(k) + b_2u(k) \\
y(k) &= c_1x_1(k) + c_2x_2(k) + du(k)
\end{aligned}
$$

In `main-prog.c` before `main()` insert `float x1[2],x2[2];`
In `main()` insert `x1[0] = x2[0] = 0;`

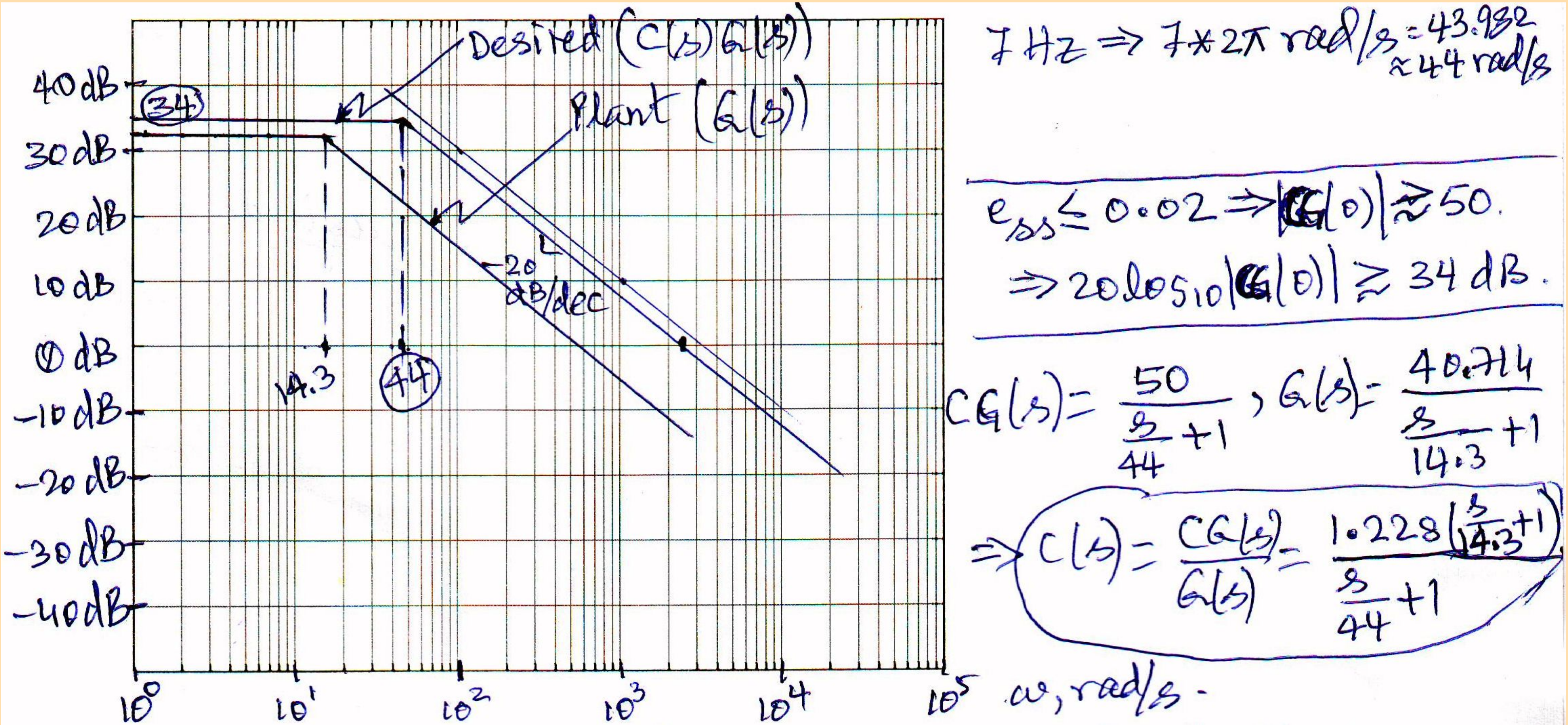
```
x1[1] = a11*x1[0] + a12*x2[0] + b1*u;
x2[1] = a21*x1[0] + a22*x2[0] + b2*u;
    y = c1*x1[0] + c2*x2[0] + d*u;
x1[0] = x1[1];
x2[0] = x2[1];
```

Part II

A solution to Problem Q2 of the pre-lab

In the lab, we will apply a sinusoidal voltage from a function generator (FG) to the dsPIC microcontroller's analog input. We will want the motor's speed to track this sinusoidal input.

Design using loop-shaping, a controller of first order such that the closed-loop system will track sinusoids of frequencies upto 7 Hz with $e_{\rm ss} \leq 2\%$ (in magnitude). For the settling time (defined as "time to enter the $x\%$ tube with the intention of remaining in it") do the best you can achieve, given the other specifications, and given that the imperfections of the plant are what they are. Hint: See EE250 lecture notes for a solution to this problem.



Note that we have achieved the tracking of sinusoids of frequencies up to 7 Hz by setting $\omega_g \approx 2500$ rad/s, which means that the closed-loop bandwidth ω_B will be approximately 2500 rad/s, which means that largest closed-loop time constrant is approximately $1/\omega_B = 0.0004$ s. So, the sampling period T will need to be $0.0004/10 = 0.00004$ s. However, our μC permits a sampling period of not less than 2 ms. So, this design is not implementable on our set up.

It may further be verified that the amplitude of the armature voltage needed to help provide such a bandwidth may well exceed the 15 V permitted by our power supply, and even the 24 V that the motor is permitted to take. So, what is the solution?

The solution is to examine what value of ω_g is permitted by $T = 0.002$ s, and to shift the above desired loop gain to the left such that its ω_g is not greater than this value. Here are the calculations of this ω_g.

$$T = 0.002\ \mathrm{s} \longrightarrow \tau \approx 0.002 \times 10 = 0.02\ \mathrm{s} \longrightarrow \omega_B \approx \frac{1}{\tau} = 50\ \mathrm{rad/s} \longrightarrow \omega_g \approx \omega_B = 50\ \mathrm{rad/s}.$$

So, the new task for the students is to find the maximum frequency sinusoids that can be tracked by a design such as the above, and to then modify the above design to track this sinusoid.

Part III

Least squares sys-id

We explain what *least squares system identification* is, not how it works.

13 Bilinear transform and Z-transform

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
g(t) \arrow[rrr, "\text{Sampling at } T_s \text{ seconds}"] \arrow[d, bend left, "\text{Laplace Transform } \mathcal{L}"] & & & g(k) \arrow[d, bend left, "\text{Z-transform } \mathcal{Z}"] \\
G(s) \arrow[u, bend left, "\text{Inverse Laplace Transform } \mathcal{L}^{-1}"] \arrow[rrr, leftrightarrow, "\text{Bilinear Transform = Tustin's method}"] & & & G(z) \arrow[u, bend left, "\text{Inverse Z-transform } \mathcal{Z}^{-1}"]
\end{tikzcd}
$$

$$s = \frac{2}{T_s}\frac{z-1}{z+1} \quad \text{or} \quad z = \frac{1+\frac{sT_s}{2}}{1-\frac{sT_s}{2}}$$

- s-domain & z-domain — fictitious domains.
- s-domain eases work with differential equations, z-domain with difference equations.

- Bilinear transform is not the only way to go $G(s) \leftrightarrow G(z)$.
- T_s limited by Nyquist sampling frequency.

14 $G(s) \longleftrightarrow G(z)$

- Consider definitions of $\mathcal{L}$ and $\mathcal{Z}$

$$G(s) = \mathcal{L}\{g(t)\} \triangleq \int_{t=0}^{\infty} g(t)e^{-st}dt$$

$$G(z) = \mathcal{Z}\{g(k)\} \triangleq \sum_{k=0}^{\infty} g(k)z^{-k}$$

- Comparison suggests $z = e^{sT_s} \Rightarrow \ln z = \ln e^{sT_s} \Rightarrow \ln z = sT_s$.

  $\therefore$ To convert $G(s)$ to $G(z)$, can substitute $s = \frac{\ln z}{T_s}$.

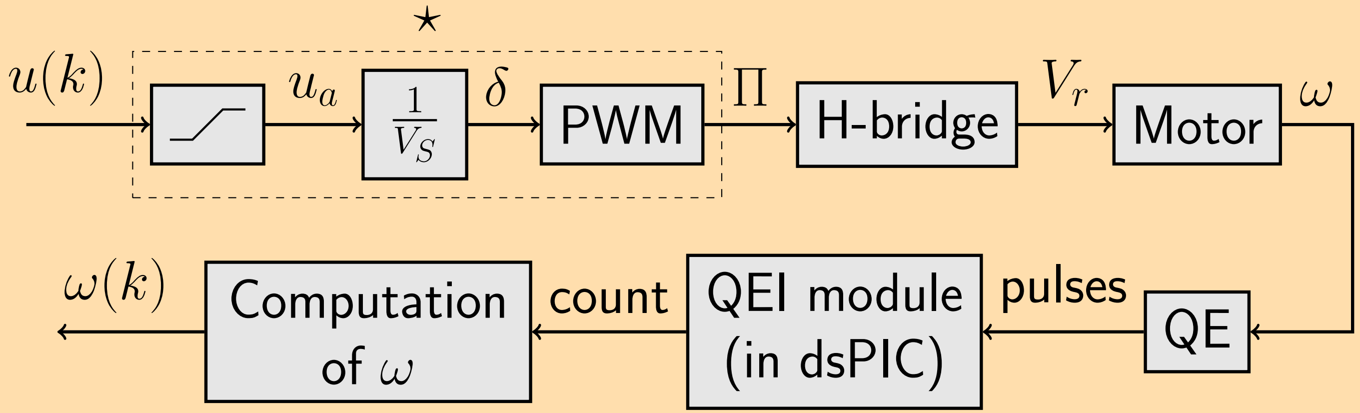
- Instead, easier to work with an approximation

$$z = e^{sT_s} = e^{\frac{sT_s}{2}} e^{\frac{sT_s}{2}} = \frac{e^{\frac{sT_s}{2}}}{e^{-\frac{sT_s}{2}}} = \frac{1 + \frac{\left(\frac{sT_s}{2}\right)}{1!} + \frac{\left(\frac{sT_s}{2}\right)^2}{2!} + \cdots}{1 + \frac{\left(-\frac{sT_s}{2}\right)}{1!} + \frac{\left(-\frac{sT_s}{2}\right)^2}{2!} + \cdots} \approx \boxed{\frac{1 + \frac{sT_s}{2}}{1 - \frac{sT_s}{2}}}$$

- This is the bilinear transform

15 How Z-transform used in our sys-id

- `terminal.log` has $u(k)$ and $\omega(k)$ obtained as



$$u(k) = u(t)|_{t=kT_s}$$

- Want a continuous-time TF $G(s)$ of plant for loop-shaping. But only have discrete-time data $u(k)$ and $\omega(k)$.

- Let TF from $u(k)$ to $\omega(k)$ be $G(z)$. That is, $G(z) = \frac{\mathcal{Z}\{\omega(k)\}}{\mathcal{Z}\{u(k)\}}$.

Then,

Step 1: Use $u(k), \omega(k)$ pairs to construct $G(z)$.

Step 2: Use bilinear transform to go from $G(z)$ to $G(s)$.

16 Step 1: Least squares sys-id (1/2)

- Let true $G(z)$ be $\dfrac{Y(z)}{U(z)} = \dfrac{b_1z^2 + b_2z + b_3}{z^3 + a_1z^2 + a_2z + a_3}$.
- Want best estimate of $b_1, b_2, b_3, a_1, a_2, a_3$.
- Cross multiply:

$$b_1z^2U(z) + b_2zU(z) + b_3U(z) = z^3Y(z) + a_1z^2Y(z) + a_2zY(z) + a_3Y(z).$$

- Multiply throughout by z^{-3}:

$$b_1z^{-1}U(z) + b_2z^{-2}U(z) + b_3z^{-3}U(z) = Y(z) + a_1z^{-1}Y(z) + a_2z^{-2}Y(z) + a_3z^{-3}Y(z).$$

- Take $\mathcal{Z}^{-1}$ to obtain difference equation

$$b_1u(k-1) + b_2u(k-2) + b_3u(k-3) = y(k) + a_1y(k-1) + a_2y(k-2) + a_3y(k-3).$$

Important property of Z-transform used:

$z^{-l}X(z) \leftrightarrow x(k-l)$ given $X(z) \leftrightarrow x(k)$.

17 Step 1: Least squares sys-id (2/2)

Consider

$$\boxed{b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) = y(k) + a_1 y(k-1) + a_2 y(k-2) + a_3 y(k-3)}$$

- Let $\sigma = [b_1 \quad b_2 \quad b_3 \quad -a_1 \quad -a_2 \quad -a_3]^\mathsf{T}$.
- Suppose we have data
  $\boxed{\{u(k), y(k)\},\ k = 0, 1, \ldots, N}$
- Problem: Find σ such that this equation holds for this data.
  I.E., find parameters of a TF that <u>fits</u> to input-output data.
- Let error in the fit be
  $$\begin{aligned}\varepsilon(k,\sigma) = b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) - y(k) \\ - a_1 y(k-1) - a_2 y(k-2) - a_3 y(k-3).\end{aligned}$$
- Modified problem: Find σ to minimize $\mathcal{J}(\sigma) \triangleq \sum_{k=0}^{N} \varepsilon^2(k,\sigma)$.
- If $\mathcal{J}(\sigma = \sigma_0) = 0$, then find best estimate $\hat{\sigma}$ of σ_0.